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ENCYSTMENT AND THE LIFE CYCLE IN THE CILIATE DIDINIUM NASUTUM[1]

By C. Dale Beers

Zoölogical Laboratory, Johns Hopkins University

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1. *The Life Cycle.*—Calkins[2] and Mast[3] made comprehensive studies concerning the life cycle of Didinium nasutum. Calkins isolated 2 didinia (*X* and *Y*) and established 5 pure lines with descendants of each. One individual from each of the 10 lines was picked out and supplied with 9 specimens of Paramecium caudatum daily. As generations passed, the fission-rate of the lines of the *X* series gradually decreased and the encystment-rate increased, the fission-rate falling to zero and the encystment-rate reaching 100% in the 131st generation. A slight increase occurred in the death-rate toward the end of the cultural cycle. The *X* series was kept through 2 additional cycles, and the *Y* series through 2 cycles of 128 and 84 generations. Calkins concluded that there are 2 types of encystment in Didinium, one which protects against environmental adversity and one which occurs periodically and independently of the environment and results in rejuvenescence, and that the passage of generations results in diminished vitality and ultimately in encystment with nuclear reorganization and rejuvenescence.

Mast found no evidence indicating that encystment occurs periodically. He kept 3 groups of lines for 1035, 831 and 850 generations without encystment or conjugation and observed no increase in encystment-rate and no decrease in fission-rate. Mast's experiments differed from Calkins' in only one respect; Mast placed no limitation on the amount of food which the didinia received. At each isolation sufficient food was added to insure a surplus at the next isolation.

The present studies were begun to ascertain the cause of the conflicting conclusions reached by Calkins and by Mast. Four lines of Didinium were established with descendants of a recently excysted individual (Clone 1) whose vigor presumably was at its maximum, and cultured in accord with Calkins' procedure, each of the 4 lines being supplied with only 9 paramecia daily. Four lines were established with descendants of the

same individual and cultured in accord with Mast's procedure, each receiving sufficient food to insure a surplus at the next isolation. Except for quantity of food, the 2 groups were cultured as nearly alike as possible. The results obtained are summarized in Table 1.

TABLE 1

THE EFFECT OF QUANTITY OF FOOD ON THE ENCYSTMENT OF PURE LINES OF DIDINIUM NASUTUM

Each group consisted of 4 pure lines cultured in parallel. The average number of daily fissions was obtained by averaging the number of daily fissions of the 4 lines. The percentage of encystment and the percentage of deaths were calculated from the number of individuals which encysted or died and the total number of individuals under observation at the time.

		GROUP 1-A SUPPLIED WITH AMPLE FOOD			GROUP 1-B FOOD LIMITED TO 9 PARAMECIA DAILY		
DAYS OF CULTURE	PERIOD ENDING	AV. NO. FISSIONS DAILY FOR PERIOD INDICATED	P.C. OF ENCYST-MENT	P.C. OF DEATHS	AV. NO. FISSIONS DAILY FOR PERIOD INDICATED	P.C. OF ENCYST-MENT	P.C. OF DEATHS
5	9/2/24	3.5	0.0	0.0	2.3	2.22	0.0
10	9/7	3.5	0.0	0.0	2.5	0.92	0.0
15	9/12	3.25	0.0	0.0	2.6	0.0	0.0
20	9/17	3.41	0.0	0.5	2.6	0.0	0.0
25	9/22	3.95	0.0	0.0	2.48	1.99	0.0
30	9/27	3.75	0.0	0.3	2.22	3.49	1.16
35	10/2	4.25	0.0	0.0	2.3	3.23	0.0
40	10/7	4.2	0.0	0.0	2.35	2.02	0.0
45	10/12	4.3	0.0	0.0	2.1	2.59	1.3
50	10/17	4.05	0.0	0.0	2.1	5.01	0.0
55	10/22	4.15	0.3	0.0	2.07	1.33	2.66
60	10/27	4.45	0.0	0.0	2.12	2.5	1.25
65	11/1	4.2	0.0	0.0	1.9	5.33	0.0
70	11/6	4.52	0.0	0.0	1.25	18.87	1.88
71	11/7	4.5	0.0	0.0	1.0	22.22	0.0
72	11/8	4.0	0.0	0.0	0.5	48.86	0.0
73	11/9	3.75	0.0	0.0	0.0	100.0	0.0

Table 1 shows that the fission-rate of the lines which received only 9 paramecia daily (Group 1-*B*) decreased as generations passed and fell to zero after 73 days of culture, that the encystment-rate gradually increased and reached 100% when the fission-rate fell to zero, and that the death-rate was higher in the last half of the period than in the first. It shows further that the fission-rate of the lines which received an abundance of food (Group 1-*A*) did not decrease as generations passed, that the encystment-rate and death-rate remained practically zero throughout the period, and that the fission-rate of this group was consistently higher than that of Group 1-*B*, the average being 3.97 fissions per day in the former with a production of 289 generations in the 73 days and 2.13 in the latter with a production of only 155 generations. The lines of Group 1-*A* were contin-

ued after the 73 day period until they had produced an average of 778 generations without encystment or conjugation. At the close of this period they were dividing as rapidly as at the beginning of culture and they showed no indications of an increase in encystment-rate.

This experiment was repeated once with didinia from Clone 1 and 4 times with didinia from 2 other clones; these repetitions gave concordant results. It was also repeated a number of times with didinia from various clones by feeding 6 paramecia per line daily instead of 9; in these experiments the fission-rate fell to zero and the encystment-rate reached 100% after only approximately 15 generations produced in approximately 11 days of culture—in place of 155 generations produced in 73 days when 9 paramecia were fed daily.

The present studies clearly indicate that the cycles observed by Calkins were the result of inadequate cultural conditions, that encystment does not normally occur periodically in Didinium as a result of internal conditions of the organism and that there is nothing in the nature of a definite life cycle in Didinium. Calkins' results and the present studies indicate that under the influence of a continued scarcity of food the initial vitality of Didinium is lessened and encystment results and that the effect of a reduced amount of food appears to be a cumulative one which gradually brings about an increase in encystment-rate and a decrease in fission-rate. Encystment under these conditions is due to an unfavorable environmental condition, lack of food, and not to depression due to the passage of generations. The mere passage of generations, as indicated by Mast's studies and by the present studies, does not result in encystment or decreased fission-rate, provided the environment is favorable and adequate. The evidence indicates that all the cysts of Didinium are of one type and are protection cysts. Whether nuclear reorganization occurs during encystment has not been ascertained. It may occur in all cysts, or in only a part of them, but nuclear reorganization during encystment is not a periodic phenomenon in Didinium.

2. *Some Factors Affecting Encystment.—Absence of food:* When vigorous, well-fed didinia belonging to the same clone are deprived of food, they divide once or twice, becoming smaller at each division; these divisions usually occur within the first 48 hours of starvation. Some of the individuals which result from these divisions encyst, while the others remain active and continue to search for food, becoming continuously smaller until after approximately a week they die of starvation. The encystments occur within the first 48 hours of starvation; individuals which do not encyst within this time usually do not encyst at all. In studies made on 3 unrelated clones, approximately half encysted and half remained active, though clones differ in reference to the percentage of individuals which encyst in the absence of food. The didinia which do not encyst when

deprived of food exhibit encystment if they are fed and again subjected to starvation, and the percentage of individuals which encyst after the second or third starvation is practically the same as after the first. No doubt the physiological condition of the didinia (conditions of fission and digestion) at the time they are deprived of food influences encystment. The fact that all the indivjduals finally encyst when food is limited to 9 or 6 paramecia daily and that only half the individuals encyst when they are suddenly deprived of food seems to be due to a difference in the rate of reduction of the quantity of food.

Effect of a recent encystment on readiness to encyst: The increase in encystment-rate and the final encystment of lines of Didinium under slightly adverse conditions (a diet of 9 paramecia per line daily) suggests that possibly under favorable and adequate environmental conditions the readiness of Didinium to encyst when unfavorable conditions intervene increases as the number of vegetative generations since encystment increases; this point was tested experimentally. The percentage of encystment under unfavorable conditions was observed in 2 groups of didinia of the same clone, one which had recently emerged from encystment and one which had passed through a number of generations since encystment. The unfavorable environmental condition to which the 2 groups were subjected was absence of food. The results indicate that didinia which have passed through 250, 500 and 750 generations since encystment, respectively, show practically the same percentage of encystment under unfavorable conditions (absence of food) as didinia which have passed through only 10–20 generations since encystment, and hence that Didinium does not become predisposed to encystment as the number of generations of culture in an adequate environment increases.

Effect of a recent conjugation on readiness to encyst: Experiments were undertaken to ascertain the effect of conjugation on the tendency to encyst The percentage of encystment in the absence of food was observed in 2 groups of didinia of the same clone, one which had recently conjugated and one which had remotely conjugated. The results indicate that didinia which have passed through 430 generations without conjugation or encystment show essentially the same percentage of encystment when deprived of food as didinia which have passed through only 5–10 generations since conjugation.

Excretion products of Paramecium: Large numbers of Paramecium caudatum were concentrated in spring water. After 3 days they were removed by filtration, and the effect of the filtrate was tested on encystment in Didinium by removing specimens to the filtrate and there depriving them of food. Of 2180 didinia thus treated, only 3 encysted, indicating that Paramecium excretes a substance that inhibits the encystment of Didinium.

Excretion products of Didinium: Large numbers of didinia were concentrated in fresh spring water and removed by filtration after 3 days. When didinia were removed to the filtrate and there deprived of food, a marked increase in the percentage of encystment resulted. About 70% of the individuals removed to the filtrate encysted, while only 45% of the individuals in spring water encysted, indicating that Didinium excretes a substance which renders the environment unsuitable for its reproduction and induces its encystment, and bearing out the observation of Mast and Ibara[4] that "rapid and extensive reproduction of didinia confined to a small space seems to favor encystment."

Hydrogen-ion concentration: The results indicate that the maximum encystment in Didinium occurs between hydrogen-ion concentrations pH 6.4 and pH 8.0; within this range the percentage of encystment is practically constant. Solutions having pH values between 8.0 and the alkaline death limit and between 6.4 and the acid death limit inhibit encystment. Solutions having pH 8.8 or higher prevent encystment completely.

Age of hay infusion culture fluid: Mast and Ibara[4] prepared a series of cultures of Didinium by removing didinia to hay infusion culture fluids of different ages, all of which contained paramecia. They observed that as the age of the fluid advanced, the percentage of Didinium cultures which encysted increased until it reached a maximum of 93.3% in fluids 6 days old; it decreased to 40% in fluids 11–12 days old. In the present studies the effect of hay infusion culture fluids of different ages on encystment was studied in the same way, except that all didinia were deprived of food, that counts were made of the actual number of individuals that encysted or remained active in each culture, that records were kept of changes in hydrogen-ion concentration, and that only half of the infusions studied were inoculated with Paramecium, the rest being inoculated with bacteria only. The results were precisely the same in hay infusions inoculated with bacteria only as in infusions inoculated with bacteria and Paramecium, provided the paramecia were removed before the didinia were put in. They indicate the following: If didinia are taken from their culture and put without food into fresh hay infusion or into hay infusion 1 or 2 days old, there is no encystment, but if the infusion is somewhat older, encystment occurs. As the age of the hay infusion increases, the percentage of encystment increases rapidly to a maximum, after which it decreases slowly. The maximum encystment occurs in hay infusions 4–6 days old. The encystment of the didinia is not dependent upon changes in the hay infusion produced by the paramecia, nor is it dependent upon changes in hydrogen-ion concentration or upon the quality of the food. The different percentages of encystment observed in hay infusions of different ages are probably due to changes produced by bacterial action. When didinia are put into fresh hay infusion, most of them are killed,

but their death is not related to the hydrogen-ion concentration of the solution.

A comprehensive treatment of this investigation will be published elsewhere.

[1] This investigation was carried out under the direction of Professor S. O. Mast, to whom the writer feels particularly indebted for valuable advice and criticism.

[2] Calkins, G. N., *J. Exp. Zoöl.*, **19**, 1915 (225–239).

[3] Mast, S. O., *Ibid.*, **23**, 1917 (335–359).

[4] Mast, S. O., and Ibara, Y., *Biol. Bull.*, **45**, 1923 (105–112).

STUDIES ON BIOCHEMICAL DIFFERENCES BETWEEN (+) AND (−) SEXES IN MUCORS

1. Tellurium Salts as Indicators of the Reduction Reaction[1]

BY SOPHIA SATINA AND A. F. BLAKESLEE

STATION FOR EXPERIMENTAL EVOLUTION, COLD SPRING HARBOR, N. Y.

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The present paper presents the results of one phase of a study of the fundamental differences between sexes. The trend of our investigations has been determined by the belief that sexual dimorphism, which is so widely distributed throughout both the plant and animal kingdoms, must have its basis in biochemical differences between the sexually distinct protoplasms. The writers realize the technical difficulties to be expected in a biochemical study of sex and for this reason feel the importance of eliminating, so far as possible, irrelevant factors such as secondary sexual characters which at best may have only an indirect connection with the primary sexual differences which are being sought. It has previously been pointed out[2] that the group of fungi included in the Mucorales are especially adapted to sex study. In the heterothallic (dioecious) Mucors the opposite sexes are so indistinguishable morphologically that it has been necessary to designate them by the signs (+) and (−). Their gametes are of equal size and the zygote in its development is apparently nourished equally by both sexes. In other forms in which the female alone nourishes the young, any differences discovered might be attributed to dimorphism in nutritive function or in other secondary sexual characters. A fuller knowledge of sexuality in the structurally simple Mucors, which are thus apparently free from the disadvantages for sex study possessed by higher forms, should help in understanding the fundamental problem of sexuality as a whole.

There are at least three requirements to be satisfied preliminary to a

critical investigation of sex. (1) The individuals studied should be shown to be in fact sexually dimorphic and not merely hermaphrodites or sex intergrades producing male or female organs in response to different environmental conditions. Extensive tests already published[2,3] or in manuscript, regarding about two thousand races of Mucors included in 30 species and 15 genera, which have been studied under varying environmental conditions and many of which have been under observation for over 20 years,[4] has lead to the belief that the heterothallic Mucors are strictly sexually dimorphic. Moreover by means of the imperfect hybridization reaction it has been possible to show that there is something common to the (+) and to the (−) sexes, respectively, throughout all the species in the group. (2) There should be a large collection of forms for study in order to show that any differences established are of sexual significance and not merely race differences unrelated to sex. We have upwards of 900 races available under cultivation. (3) There should be knowledge of the strengths of sexual activity of the forms investigated in order to be able to correlate any sexual differences with such sexual activity. Grading of sexual strength has been done for practically all of the several hundred races in our collection.

A number of investigators have realized the value of a study of sexual differences in Mucors. Thus Korpatchewska[5] found that the (+) race of Mucor hiemalis assimilated maltose as the source of carbon better than the (−) race while the reverse was true in regard to sucrose. Breslauer[6] found a better assimilation of carbon by the (+) race of M. hiemalis and concluded also that in this species the enzyme sucrase is formed only by the (+) race. Her work was confirmed by Lendner.[7]

Working with Phycomyces nitens, Lindner[8,9] found that it is the (−) race which is better able to assimilate sugar while Orban[10] with the same species reached an exactly opposite conclusion. Dunn,[11] working with a single (+) and a single (−) race of Rhizopus obtained from our laboratory, found pronounced physiological differences in their response to changes in the substratum upon which they were grown.

Kostytchew and Eliasberg[12] working with Mucor racemosus found that oxidation processes proceed more actively in the (+) than in the (−) race but that only the (−) race contains invertase and is therefore alone able to invert cane sugar.

In regard to the foregoing investigations it may be pointed out that in only one case do we know that the (+) and (−) races studied correspond to the sexes we have designated by these signs and in consequence the results obtained cannot be compared. A more fundamental criticism of these investigations, however, deals with the fact that, so far as any evidence is offered, none of these biochemical studies involved the investigation of more than a single (+) and a single (−) race. The assumption

has been made that the various races of a given sex do not differ in physiological reactions. Such an assumption our present paper will show is unfounded and therefore the investigations cited can have little significance so far as purely sexual differences are concerned. In consequence in our studies of sexual differences, we have felt the necessity of investigating at first extensively a large number of (+) and (−) races, rather than intensively a single pair. It is our purpose, therefore, to run through a series of easily applied tests on a representative sample of sexual races with the hope of discovering a clue to some biochemical differences between the two sexes which are constant throughout the whole group.

The present paper gives the results of tests of the reduction reaction using the "vital reaction" of Gosio,[13] which involves the capacity of the living cell to absorb salts of Te and Se and reduce them to these elements—black in the case of the former and red in the latter. For the details of the technique involved, reference may be made to the original paper of Gosio. The reaction is a delicate one and, for successful results, attention must be paid to a number of factors such as the type of salts used and their concentration, the nutrient medium and its acidity, temperature, light, age at which cultures are recorded, the quantity of spores used in inoculation, etc. Na_2TeO_3 and K_2TeO_3 were more satisfactory than the corresponding selenites since they are more resistant to heat in sterilization, are less toxic and the black precipitate which they give at reduction is more readily graded than the red Se. The optimum concentrations for the tellurites seemed to be from 1:5000 to 1:10,000. Distinct pigmentation was evident in the majority of species at a concentration of 1:25,000, whereas a concentration of as much as 1:2000 distinctly checked growth of the cultures. With the selenites a concentration of 1:25,000 seemed to be best. As a source of carbon, dextrose and maltose were the best among the carbohydrates tested and accordingly our standard agar nutrient No. 230 (consisting of 2% each of agar, dextrose and malt extract with addition of 0.1% peptone) was used as a substratum. In order to avoid differences in the density of growth which cause changes in the apparent strength of the reaction, the number of spores were estimated by aid of a blood counting apparatus and their number equalized by proper dilutions of the different spore suspensions. Usually a concentration of 1000 to 1200 spores in 1/400 c. mm. was used in inoculations. The cultures were grown side by side in large Petri dishes in divisions of equal area marked out by lines on the bottom of the dishes. The time necessary to show a differential reaction varied for the different species from 16 hours to 2 or 3 days. Due possibly to differences in permeability of the cells, there were differences observed in the speed of the reaction within a given species. Consequently the races of each species were recorded after their optimum period of growth at a constant temperature of 26° C. Grading

TABLE 1

REDUCTION REACTION IN TWO SPECIES OF MUCORS ON NUTRIENT NO. 230 (2% EACH OF AGAR, MALT AND DEXTROSE PLUS 0.1% PEPTONE) WITH THE ADDITION OF $Na_2Te\,O_3$ IN PROPORTION OF 1:5000. CULTURES KEPT AT 26° C.

A. ABSIDIA BLAKESLEEANA LENDNER. 40 RACES, 20 (+), 17 (−), 3 NEUTRALS (N). CULTURES GRADED AFTER 96 HOURS.

RACE NO.	SEX	COLOR TOP RECORD WHITE	BLACK
202	(+)	7	93
355	(+)	7	93
345	(+)	7	93
356	(+)	7	93
347	(+)	7	93
353	(+)	10	90
201	(+)	10	90
350	(+)	10	90
364	(+)	10	90
348	(+)	10	90
716	(+)	10	90
358	(+)	10	90
349	(−)	10	90
371	(−)	10	90
354	(N)	10	90
357	(+)	15	85
366	(+)	15	85
715	(−)	15	85
367	(−)	15	85
368	(N)	15	85
351	(+)	20	80
359	(+)	20	80
203	(+)	20	80
713	(−)	20	80
638	(−)	20	80
571	(−)	20	80
365	(−)	20	80
362	(−)	20	80
811*	(−)	20	78
363*	(−)	21	77
711*	(−)	25	73
714	(+)	30	70
369	(+)	30	70
346	(−)	30	70
712	(−)	30	70
360	(N)	30	70
370	(+)	40	60
572	(−)	40	60
352	(−)	40	60
717	(−)	50	50

B. CIRCINELLA SPINOSA. 42 RACES, 29 (+), 12 (−), 1 NEUTRAL (N). CULTURES GRADED AFTER 48 HOURS. IN MATCHING COLORS OF CULTURES 1% ORANGE WAS USED IN EACH CASE.

RACE NO.	SEX	COLOR TOP RECORD WHITE	BLACK
788	(+)	5	94
789	(+)	5	94
418	(+)	7	92
563	(+)	7	92
426	(+)	7	92
559	(+)	7	92
425	(+)	7	92
568	(+)	7	92
785	(+)	7	92
554	(+)	7	92
790	(+)	7	92
430	(+)	10	89
780	(+)	10	89
413	(−)	10	89
432	(+)	12	87
431	(+)	12	87
557	(−)	12	87
813	(N)	12	87
555	(+)	14	85
427	(+)	14	85
791	(+)	14	85
556	(+)	14	85
421	(+)	14	85
794	(−)	14	85
781	(−)	14	85
429	(−)	14	85
414	(−)	14	85
417	(+)	16	83
564	(+)	16	83
784	(+)	16	83
424	(+)	16	83
792	(+)	16	83
553	(+)	16	83
786	(+)	16	83
422	(+)	16	83
558	(+)	16	83
782	(−)	16	83
419	(−)	19	80
416	(−)	19	80
562	(−)	19	80
428	(−)	30	69
415	(−)	30	69

* Cultures of races marked with an asterisk (*) required the addition of 2% orange in color top in grading cultures.

was done by aid of the Bradley color top, using black and white sectors. Rarely a small amount of orange was necessary in matching the color of the fungus tested. In order to check the development of the fungi the tested cultures were always examined microscopically. It was further desirable to determine how the particles of tellurium and selenium were distributed within the cells. Without going into details, it may be mentioned that the grains of Te were visible in spores as soon as they began to germinate and after germinating they were usually strongly pigmented. In fully formed hyphae, the Te was very unequally distributed. Some of them were more strongly pigmented near the growing tips, whereas in others the pigmentation was stronger in the older parts. It was impossible, however, to determine in most cases in which part of the cell the strongest reduction had taken place. Too strong concentration of the salts showed its toxicity by causing the hyphae to form short cells, some of them to rupture at their tips and extrude irregular masses of colored plasm.

In a series of preliminary experiments, there were tested a total of 28 races in 9 species and 5 genera and in every case the (+) races showed a greater reduction than the (−) races of the same species as indicated by the greater percentage of black needed in matching the color of the cultures. It seemed, therefore, that we had a lead as to a constant difference between the (+) and (−) sexes. Despite these results, however, although they were based upon a much larger number of races than had ever before been tested biochemically, later tests with a still larger number of races showed that the (+) race is not invariably a stronger reducer than the (−) race with which it is capable of producing sexual spores.

Table 1, *A* and *B*, shows the detailed tests for Absidia Blakesleeana and Circinella spinosa in which all the races available were investigated. Although in each species some (−) races are stronger reducers than some (+) races and vice versa, it is evident that there are more (+) races among the strong reducers and more (−) races among the weak reducers. Similar tables have been made for the other species tested. The results are summarized in table 2. A total of 264 races (131 (+), 128 (−) and 5 neutral) were investigated. In all, 6 genera and 13 species are included, and the collection may be considered fairly representative of the group of dioecious Mucors. It is evident that in all the species, except Helicostylum in which the reduction shown by the two sexes is equal, the (+) races have a higher average power of reduction than the (−) races as measured by the amount of black needed in matching the cultures. The total averages are 87.4 black for the (+) against 80.3 for the (−). When the strongest and weakest reducers are listed separately, it will be seen that (+) races predominate among the strong reducers [63 (+) to 27 (−)] and (−) races predominate among the weak reducers [36 (−) to 12 (+)].

It is perhaps significant that as yet we have found no species in which

(−) races show, on the average, a stronger power of reduction than the (+) races.

What significance is to be attached to the evident greater average reducing capacity of the (+) race cannot be told as yet. It is conceivable that the sex differential directly influences the reducing power of the living cells but that there are other non-sexual factors which are also of influence and in respect to which the races of either sex differ. Tests with other reagents upon the same races may throw light upon this question.

TABLE 2

SUMMARY OF TESTS WITH $Na_2Te\ O_3$, SHOWING FOR EACH SPECIES THE NUMBER OF (+), (−) AND NEUTRAL (N) RACES TESTED, THE NUMBER OF (+) AND (−) RACES AMONG THE STRONGEST AND AMONG THE WEAKEST REDUCERS, AND THE AVERAGE STRENGTH OF REDUCTION SHOWN BY THE (+) AND (−) RACES MEASURED IN PERCENTAGES OF BLACK NEEDED TO MATCH CULTURES WITH A COLOR TOP

SPECIES	STRONGEST REDUCERS (+)	STRONGEST REDUCERS (−)	WEAKEST REDUCERS (+)	WEAKEST REDUCERS (−)	TOTAL NUMBER OF RACES (+)	TOTAL NUMBER OF RACES (−)	TOTAL NUMBER OF RACES N	AVERAGE PER CENT BLACK (+)	AVERAGE PER CENT BLACK (−)
Absidia Blakesleeana	12	2	1	3	20	17	3	85.3	75.2
Absidia glauca	2	1	0	2	4	4	–	82.5	76.2
Absidia "Whorled"	7	4	5	6	14	16	1	88.0	85.8
Circinella spinosa	11	0	0	5	29	12	1	87.5	81.4
Circinella umbellata	1	0	0	1	1	1	–	88.0	67.0
Cunninghamella Bertholletiae	7	10	1	7	12	23	–	84.0	80.0
Helicostylum	4	1	2	1	8	4	–	81.0	81.0
Syncephalastrum	10	5	2	6	28	28	–	85.2	80.8
Mucor griseo-cyanus	1	0	0	1	1	1	–	90.0	80.0
Mucor IV	5	4	1	1	11	19	–	92.1	91.4
Mucor VII	1	0	0	1	1	1	–	88.0	84.0
Mucor D	1	0	0	1	1	1	–	89.0	81.0
Mucor N	1	0	0	1	1	1	–	95.0	80.0
Totals	63	27	12	36	131	128	5	87.4	80.3

We have attempted to correlate the variation in power of reduction with several of the more obvious racial differences and have reached the following conclusions.

1. The power of reduction has no direct relation to the sexual strength of the races. Sexual strength is calculated from the relative number of sexual spores which are formed when the given race is grown in contact with appropriate tester races.[2,3] Tables are available from previous studies giving for each (+) and (−) race an average grade based on individual ratings on a scale of 0 to 4. Mucors of both strong and weak sexual grade were found among both strong and weak reducers. Thus in Circinella (Table 1-*B*) the weak reducers (−) 416 and (−) 562 are graded 2.5 and 1.0, respectively, and the strong reducers (+) 559 and (+) 426 are graded 0.5 and 2.0.

2. The power of reduction has no direct relation to the vegetative vigor. Thus in Syncephalastrum, out of 8 of the strongest reducers, 6 were of vigorous growth, 1 of medium growth and 1 of very weak growth. On the other hand, out of 8 of the weakest reducers, 4 were vigorous, 3 of average and 1 of weak growth. A similar conclusion is reached if vigor is measured by the development of sporangia.

3. The reduction is not directly related to differences in the size of spores characteristic of races in certain species such as Absidia Blakesleeana.

4. The failure to germinate of some of the spores used for inoculation is not responsible for weak reduction as shown by germination studies in hanging drop cultures.

The experiments with salts of Te and Se have established the variability of races of the same sex in regard to their ability to reduce these compounds. They indicate that on the average the (+) sex has a greater power of reduction than the (−); although, due perhaps to non-sexual factors in respect to which the races differ, there is as yet no simple and direct relation evident between this power possessed by a given race and its sex. They demonstrate the absolute necessity of basing any conclusions that may be drawn in regard to sex differences upon the behavior of a relatively large number of individual races.

[1] Paper presented in brief before the Physiological Section of the Botanical Society of America, December 29, 1924. This investigation was carried on under a grant from the Committee for Research on Sex Problems of the National Research Council for whose support the authors gratefully acknowledge their indebtedness.

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TETRAPLOIDY IN CITRUS[1]

BY HOWARD B. FROST

CITRUS EXPERIMENT STATION, RIVERSIDE, CALIFORNIA

Citrus seeds often contain adventitious embryos, produced, usually subsequent to fertilization of the egg, by proliferation of nucellar cells adjoining the embryo sac.[2] Seedlings developed from these embryos are therefore entirely "maternal" in their inheritance, and in interspecific crosses they can usually be readily distinguished, even in advance of fruiting, from the hybrids that result from fecundation. Among the Citrus progeny from cross-pollination that are now growing in the breeding orchard at the Citrus Experiment Station, the proportion of "apogamic" individuals usually ranges about from 20 to 85 per cent of the whole number of seedlings of a seed-parent variety. Besides the ordinary apogamic progeny, another form that shows no characters of the pollen parent, and therefore is obviously produced asexually, has frequently been found. This form is strikingly distinct from the ordinary apogamic progeny, and its differential characters are so similar in the various species and botanical varieties concerned that a single mode of origin of this aberrant form is strongly suggested. It is somewhat slower in growth than the normal apogamic progeny of the same parentage, and is about one or two years later in its first flowering. It has stout shoots and thorns, and broad, thick leaves. It has been designated "thick-leaved."

The haploid number of chromosomes in three horticultural varieties of Citrus, or in their apparently identical seed progeny produced apogamically, has recently been found to be 9.[3] This is illustrated by figure 1,[4] which shows the second metaphase in a pollen mother cell of normal apogamic progeny of Paper Rind (St. Michael) orange (*Citrus sinensis* Osbeck) from an interspecific cross. There are clearly 9 chromosomes in each group.

Few flowers have as yet been produced by the thick-leaved trees, but chromosome counts have been obtained from pollen mother cells of several buds of a thick-leaved sib of the diploid tree just mentioned. At the second metaphase and anaphase the number of chromosomes in each group approximates 18 (fig. 2). The number seems highly variable, and counts of 16, 17, 18, 19 and 20 have been obtained. Evidently, as in tetraploid Datura,[5] non-disjunction occurs very frequently at the first metaphase. Polar views at the first metaphase indicate that the chromosomes are paired as usual, and that part or all of the bivalents are often more or less closely associated in quadrivalent groups. In the small number of cases so far observed, the formation of quadrivalents is much less general than in tetraploid Datura, and is obviously secondary to or-

dinary pairing; in the formation of quadrivalents this tetraploid Citrus seems to be intermediate between tetraploid Datura and tetraploid Primula.[4] Figure 3 shows an unusually clear *lateral* view of the first metaphase, in which all of the 18 bivalents seem to be distinguishable. The pollen mother cells and pollen tetrads are considerably larger in the tetraploid form than in the diploid.

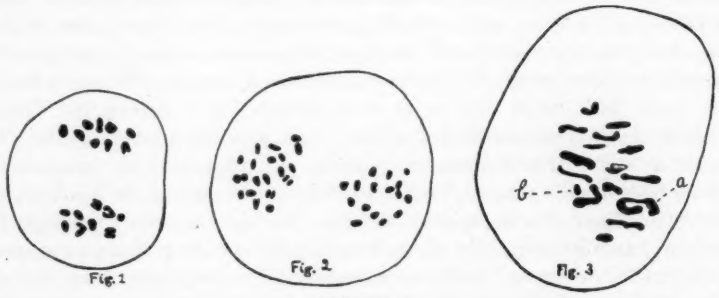

FIGURE 1

Normal (diploid) apogamic seedling progeny of Paper Rind orange. Metaphase of second meiotic division in pollen mother cell; 9 chromosomes, often distinctly split, in each group.

FIGURE 2

Tetraploid apogamic sib of the tree represented by figure 1. Second metaphase; 16 chromosomes in one group and 20 in the other; chromosomes seldom clearly split, but one widely so.

FIGURE 3

Same tree as figure 2. Lateral view of late metaphase of first meiotic division, evidently showing 18 bivalent chromosomes. At "a," two bivalents joined by a strand of chromatin; at "b," a bivalent not yet oriented on the spindle.

In the diploid tree mentioned above the pollen-tetrad stage usually appears normal, but occasionally one or two microcytes are present (for example, in 2 out of 50 random tetrads from one bud). In the tetraploid sib, on the other hand, microcytes are conspicuously numerous; counts of 50 tetrads each for two buds show 28 and 39 tetrads, respectively, with from one to four microcytes. Apparently detachment (elimination) of chromosomes is much more frequent in this tree than in tetraploid Datura.[5]

A few division figures have also been obtained for thick-leaved progeny of Lisbon lemon (*C. Limonia* Osbeck), and several approximate counts indicate that the doubled number of chromosomes is present. It may be confidently anticipated therefore, in view of the general somatic similarity of the thick-leaved forms, that they will all prove to be tetraploid.

The thick-leaved type has been produced by 12 horticultural varieties of Citrus, representing 4 species and 2 botanical varieties, and sometimes to the extent of several per cent of the whole number of progeny. It is

possible that "islands" of tetraploid tissue are common in the parent trees, so that not all tetraploid seedlings represent distinct cases of doubling of chromosome number. Chimeras combining the normal and thick-leaved forms have not been found in our cultures, however, and C. S. Pomeroy states that, in the bud-variation work on Citrus directed by A. D. Shamel, the thick-leaved form has not been observed. The lack of thick-leaved bud-variation forms might, of course, be due to the slower growth of tetraploid cells, which might keep them from multiplying sufficiently to dominate any apical meristem, rather than to failure of tetraploidy to originate outside the nucellus. In many cases, however, a thick-leaved seedling has developed from a seed giving two or more non-hybrids altogether, and in these cases the other apogamic progeny from the same seeds have almost always been normals. It is therefore probable that doubling of the chromosome number occurs with especial frequency in the nucellar tissue, at least in those cells from which embryos are about to develop, or else in the very young embryos. So far as seed reproduction is concerned, tetraploidy seems to originate far more often in several Citrus species, than does either tetraploidy or triploidy in any other species in which its origin has been observed.

The thick-leaved forms of Citrus do not so far give promise of direct horticultural value. It may be feasible, however, to produce valuable new forms, triploids or modified triploids, by crossing with ordinary diploid varieties. Triploids would probably be practically seedless.

[1] Paper No. 130, University of California, Graduate School of Tropical Agriculture and Citrus Experiment Station, Riverside, California.

[2] Strasburger, E., *Jenaische Zeitschr. Naturw.*, **12**, 654 (1878), and *Jahrb. wissensch. Bot.*, **44**, 482–555 (1907); Webber, H. J., *Science*, n. s., **11**, 308 (1900).

[3] Frost, H. B., *Wash. Acad. Sci. Jour.*, **15**, 1–3 (1925).

[4] The figures were drawn with the aid of a camera lucida, at a magnification of about 3050 diameters, and are reduced one-half in reproduction. The slides were prepared by Belling's method (*Amer. Nat.*, **55**, 573–574 (1921)), in the blooming season of 1925. In figure 3, part of the variation in size among the separating chromosomes may be due to variable swelling produced by acetic acid.

[5] Belling, J., and Blakeslee, A. F., *Amer. Nat.*, **58**, 60–70 (1924).

DEVIATION FROM THE REGULAR AS AN ART PRINCIPLE

By C. E. Seashore and Milton Metfessel

University of Iowa

Read before the Academy April 21, 1925

The background for this paper lies in a series of studies of regularity, rigidity, and precision in sound waves. During this year, in our laboratory Dr. Simon has measured variability in the wave-lengths of consecutive sound waves in music and speech. Mr. Kwalwasser has continued studies begun by Schoen on laws of the vibrato. Mr. Robert Sea shore has studied temporal precision in motor rhythms. Dr. Travis has studied the nature of the rigidity in the voice of stutterers under emotional stress. Mr. Metfessel has aimed to isolate certain constant tendencies in the expression of emotion in folk songs.

The principle involved is a well recognized theory of art. The new feature is the objectifying of it in terms of quantitative measurements. In music and speech, pure tone, true pitch, exact intonation, perfect harmony, rigid rhythm, even touch, precise time play a relatively small role. They are mainly points of orientation for art and nature. The unlimited resources for vocal and instrumental art lie in artistic deviation from the pure, the true, the exact, the perfect, the rigid, the even and the precise. This deviation from the exact is, on the whole, the medium for the creation of the beautiful—for the conveying of emotion. That is the secret of the plasticity of art. The exact is cold, restricted, and unemotional; and, however beautiful, in itself soon palls upon us.

The variation from the exact which is due to incapacity for rendering the exact is, on the whole, ugly. The artist who is to vary effectively from the exact must know the exact and must have mastered its attainment before his emotion can express itself adequately through a sort of flirtation with it.

The aim of our paper is to show, by one example how by the process of photographing sound waves, we obtain adequate data for an objective portrayal or description of the expression of emotion in music; and that this description as a rule becomes a statement of the character and the extent of deviations from the exact in each and every attribute of sound. Such expression of emotion is the language of emotion in music.

There are four elemental attributes of sounds; pitch, duration, intensity, and extensity. Extensity may be combined with pitch, with which it varies inversely. This leaves us then only three elemental factors to deal with—pitch, time, and intensity.

Of the complex traits of sounds Volume may be expressed in intensity, extensity, and the re-duplication of these. Rhythm is made up of time

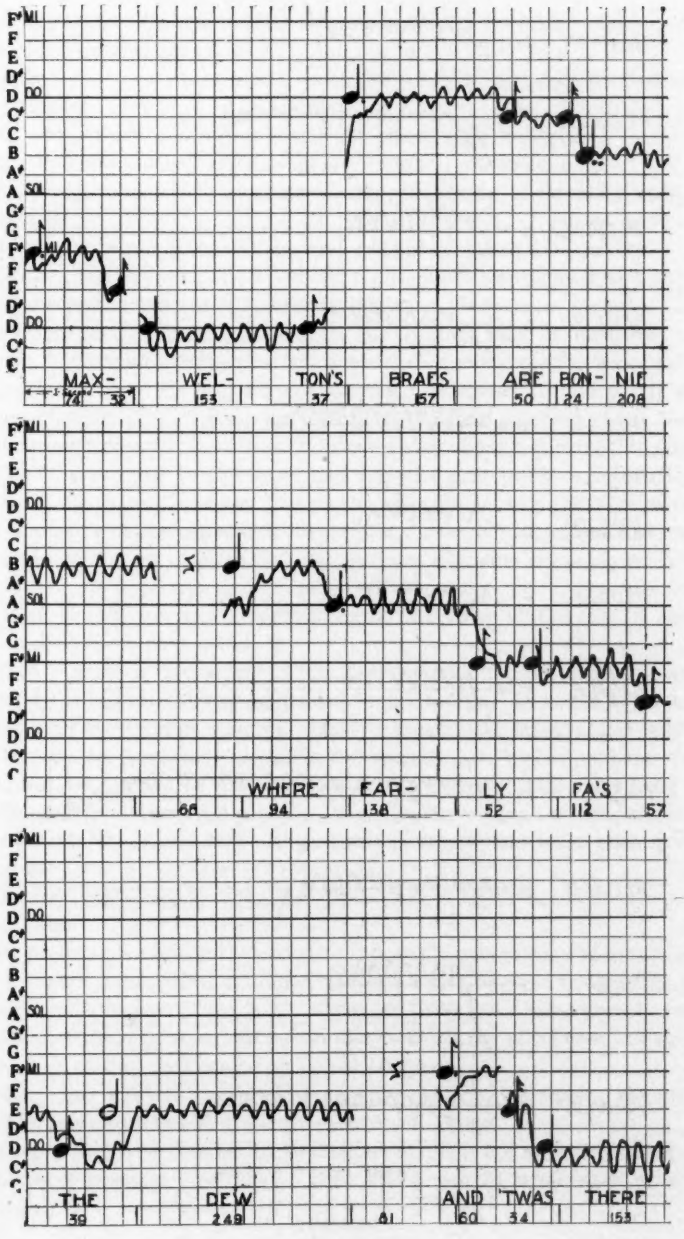
MAX- WEL- TON'S BRAES ARE BON- NIE
78 32 153 37 157 50 24 208
WHERE EAR- LY FA'S
68 94 138 52 112 57
THE DEW AND 'TWAS THERE
39 249 81 60 34 153

FIGURE 1

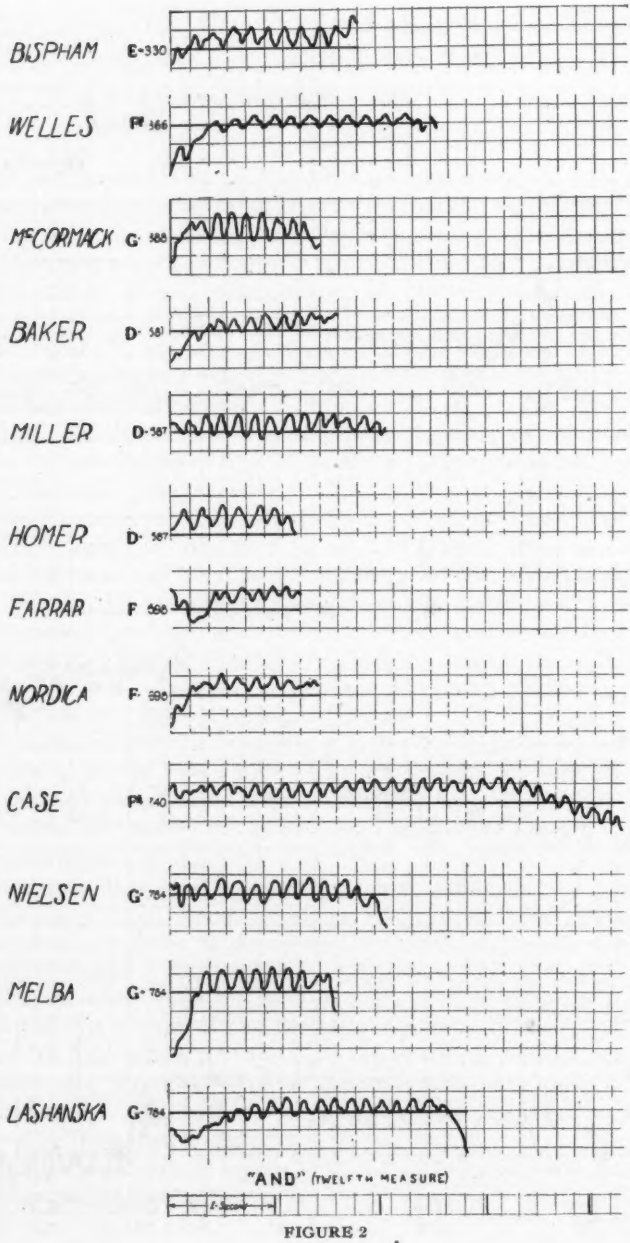
BISPHAM
WELLES
McCORMACK
BAKER
MILLER
HOMER
FARRAR
NORDICA
CASE
NIELSEN
MELBA
LASHANSKA
"AND" (TWELFTH MEASURE)
1-Second

FIGURE 2

and intensity. Consonance is a combination of pitches. Timbre is a complex of pitches.

Therefore, to interpret sound waves for practical purposes we deal with four factors:

1. Frequency, which gives us all the phenomena of pitch.
2. Duration, which gives us all the temporal phenomena.
3. Amplitude, which gives us all the phenomena of intensity or loudness, and
4. Form, which gives all the phenomena of timbre or tone qua ity.

In the present example, two of these phases are represented by the musical score; the phenomena of pitch and the phenomena of time. The other two factors, namely, intensity and timbre, may be measured and shown in a similar manner, but are not included in the present score partly because too many data in a single score would clutter it, and partly also because at the present stage our interests have not centered about these, and our technique for them is not fully adequate, though quite within reach.

Let us now turn to the score and consider first pitch or intonation.* None of the singers hold any true pitch. The flutter in pitch is the *vibrato*. On the basis of measurements like these we define a good vibrato as a periodic oscillation of pitch, taking the form of a sine curve representing an amplitude approximately a half-tone interval in pitch, synchronizing with an equally perceptible amplitude for intensity at the rate of about seven oscillations per second.

Such vibrato is one aspect of the expression of tender emotion. It is a highly refined trembling expressing emotionality. It is not to be confused with the tremolo which is usually exaggerated, labored and ugly.

Observe the continuity of this in the song by Mr. Welles, and the similarity of his vibrato to that of the other 11 artists. The verse sung by Welles was photographed directly from singing without accompaniment. The sample tone from other singers was photographed from phonograph records.

Unemotional singling lacks this grace, and is either exact or erratic in intonation.

By present photographic methods it is possible to organize the foundation for a science of countless emotional factors, and thus pave the way for the theory and practice of the art. Many of these laws have already been established, but the most significant fact today is the proof that emotional expression of this kind can be measured adequately.

Note other matters of intonation; e.g., the attack, the release, the glide, the tie—each a means for the expression of emotion.

Among the most interesting deviations of pitch are the corrections for illusions of hearing. To make a tone sound right it must often be sung wrong.

Notice also observable personal traits; e.g., the tendency for Case to linger on her high flute notes; or Melba's characteristic form of attack for a note of this kind.

Turn for just a moment to the temporal phenomena—time, tempo, rhythm—all expressed in the numerical notation and the spacing at the bottom of the score. Here we find again the "flirtation" with the exact —one moment hitting time with precision, then a dash of defiance, a wistful approach, an assertion of freedom which is but the affirmation of law— that grace, that boldness, that richness of charm which stirs in us the emotion that the artist feels.

* In presenting this paper before the National Academy, the speaker exhibited a complete score of "Annie Laurie" as sung by Laurel Welles and, for comparison, one note in this selection as sung by ten other artists. Figure 1 is a sample of the former and Figure 2 of the latter. Musical notes are designated by letters at the left and, in Figure 2, by vibration frequency. Each vertical space denotes a half-tone in pitch and each horizontal tone 0.17 sec. in time.

THE RÔLE OF MENTAL MEASUREMENT IN THE DISCOVERY AND MOTIVATION OF THE GIFTED STUDENT

By Carl E. Seashore

University of Iowa

Read before the Academy, April 27, 1925

The introduction of psychological measurements has revealed three features of individual differences with such striking force that the educator must take notice of them to the extent of radical changes in the educational procedure. These are: (1) the magnitude, (2) the fixity, and (3) the intricacy in organization of varieties of individual differences.

It is a general rule that the magnitude of individual differences varies with the specificness of the feature measured. For example, two students may seem to vary in college work as one to two in the general power of memory, but when we test them on a specific type of memory, such as memory for geometrical form, the difference may prove to be 5, 10 or 20 times as great; and the same person may have 5, 10 or 20 times as good memory for geometrical form as he has for tonal quality, or *vice versa*. Such differences may represent vital conditions for success; as, for example, on the one hand in graphic arts, on the other in music. The same principle applies to countless capacities in mathematics, natural science, philosophy or literature.

Our best measures of such fixity are appearing in the realms of the psychophysic capacities, motor personal equations, and types of mental

alertness. The sense of pitch and the sense of rhythm, for example, may be measured early in childhood and are found to remain constant throughout life except for deterioration, normal or pathological, despite most elaborate education and special training. A boy who is quick and accurate is likely to be the man who is quick and accurate in any particular type of motor process. As our measures of intelligence are gradually improved, we find more and more evidence that the intelligence quotient of 75, 100, or 125 tends to remain fairly fixed throughout life, with or without extensive education of the individual.

Measurement, likewise, reveals intricacy upon intricacy in the order of mental organization. Philosophers in the past have spoken of the psychophysic organism as a microcosm, but perhaps only in the same sense that the peasant may regard the starry heavens as macrocosm. The mental telescope and spectroscope, if I might use that figure, are beginning to take form. The photographing of the minutest detail in the expression of emotion, in song or speech, lays bare for measurement hierarchies upon hierarchies of orderly factors.

The introduction of measurement into any new field results (1) in the repeated use and progressive extension into varieties of measurements, (2) in the recognition of fundamental principles involved and the building of a science, and (3) in the development of a point of view based upon these facts and tending to crystallize in the form of a common sense. The inceptions of these three stages are now being illustrated in the introduction of measurement for the guidance and education of the individual. Let me exemplify by what is going on at the college level *for the progressive elimination, selection and motivation of the individual.*

The first step is the introduction of a new type of college qualifying examination near the end of the senior year in the high school. This examination is an outgrowth of the mental test movement and it is interesting to see how the more rigid technique of the early psychophysicists and later laboratory methods are making their impressions upon these examinations and finding new expression, particularly in the statistical method and in the setting of the problem. These examinations may be of the objective type, comprehensive, fair and impersonal. There is a tendency to divide them into two equally important parts—training and aptitude. The former serves the function of the traditional college entrance examination, testing the quality of training, the latter is built on the psychology of individual differences and aims to measure natural endowment, in aptitude or capacity for observation, and action of the types required for success in college work. It incidentally yields also an intelligence quotient. These examinations are prepared with great care and their reliability and validity are studied with the aim to improve them on the basis of experience. In one state we are now in the third year of following from 1500 to 2000

students for each year determining the correlation between this prediction made in the high school and college achievement. The preliminary findings are very encouraging. The high school senior may through this examination obtain an invaluable forecast of probable success in college. The effect of approaching this subject in a scientific attitude and with rapidly forming scientific technique is the main point I wish to stress.

The giving out of the results of the examination will be accompanied by a service of carefully prepared material for interpretation and guidance. This service will be a far-reaching leverage for the arousing in the community the idea that it makes a difference whether a youth has or has not a college brain and will encourage a movement to make plans for a college career on the basis of facts about the individual. This will discourage the unfit and encourage the fit, which is our main objective, before the great risk of the college venture has been taken. It will improve our college timber and reduce college failures. A logical sequel to this will be the development of a reasonable guidance of those found unfit for college, revealing to them new outlets for which they may be thoroughly fit.

The next stage of this sorting process takes place at college entrance. Here every freshman is met by an important diagnostic sifting in and for each freshman subject by what is known as a placement examination. The first two class periods in each freshman subject are devoted to this examination. In English, for example, the first hour is devoted to a highly standardized test of training for English. The second hour is devoted to an analysis and measurement of Language ability and particularly to aptitude for command of English. Both examinations are of the objective type and may be scored over night, so that at the opening of the third day all of the students in English are classified in the order of percentile rank on two scores, (1) training, and (2) aptitude for work in English.

The same is done for all the other freshman subjects. By this means each department will have at the very beginning of the course valid basis for the organization of sections or laboratory units on the basis of fitness, so that it may live up to the new educational slogan which has come out of the technical study of individual differences, namely, *to keep each of the students busy at his highest natural level of successful achievement.*

These measures from all departments, assembled in the dean's office, together with the report of the qualifying examination from the high school, furnish an invaluable profile for personnel work with the students. The dean has a right to expect achievement on the basis of capacity and this is a radically new idea in American education.

The preliminary returns from twelve institutions from the first semester of this year give a correlation of 0.65 between prediction through placement examination and achievement in terms of semester grades for all of

the freshman subjects investigated. This is a rather extraordinary achievement. Our statistician on this subject, Mr. Stoddard, authorizes this statement:

"The predicative value of the placement examination is higher for its *particular subject* than a reliable group intelligence examination; it is much higher than that yielded by a survey test of high school content material; and much higher also than a composite testing program of intelligence, reading comprehension, and high school content."

In another series of experiments, it has been shown that the predicative value of a placement examination of this kind is higher than that of the rating that can be given by the instructor on the basis of experience for one semester in an unsegregated group. This salvage of the good student at the very beginning saves him from a semester of floundering. Pedagogically the upshot of this is the inevitable practice of grouping students, assigning work, and measuring achievement on the basis of ability. In the elementary courses this generally takes the form of a sectioning on the basis of ability, where the work is done in class, or freeing the student for individual progress in laboratory and project units. Thus the man who comes with good heredity to freshman English from a good home, where he has used good English, read good books, has been trained in a good school and has acquired good habits of study and will to work, is for the first time charged with the responsibility of investing all his talents, and the man at the other extreme of endowment and training for the first time is treated with respect and trained efficiently at his own level. They have nothing in common.

The third step which is coming upon the horizon is the application of scientific technique to the determination of our aims and objectives in a course and the setting of valid tests of achievement. The cutting of the leash which has traditionally held the good student to the level of mediocrity, forces the enlargement of the scope of reasonable assignment for the good student in each course. A striking example of the result of such enlargement of scope and new motivation is shown by the fact that under these conditions with the good student a passing mark has been lowered numerically from, e.g., the traditional passing point of 70 to somewhere between 25 and 30. That is, assuming that we have the same standard of achievement for passing which we had ten years ago under the old method when we passed the student who earned 70% on the examination, we now pass all who make a record of about 25% in an objective examination in which all who make A or B (the highest quartile) must pass above 75%. This is no lowering of standards, but comes from setting the good student free for distinguished achievement in proportion to his capacity.

ARITHMETIZATION IN THE HISTORY OF MATHEMATICS

BY G. A. MILLER

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ILLINOIS

Communicated August 7, 1925

Arithmetization is doubtlessly the most dominating principle in the history of mathematics and hence this history can be studied most effectively by keeping this principle in the foreground. In particular, the ancient Greeks developed a geometrical algebra which the later Greeks themselves began to arithmetize as may be seen in the *Arithmetica* of Diophantus. The Indian mathematicians extended this arithmetization, especially as regards the use of negative numbers, while the Arabs brought the algebraic equation into greater prominence; in particular, by reducing the Archimedean problem to the solution of a cubic equation. On the other hand, they extended the geometric algebra of the Greeks in their successful efforts to find one root of this equation geometrically in the important case when it has a positive root. The extension of the Greek geometrical algebra thus established was later arithmetized by Italian mathematicians during the sixteenth century by the discovery of a general formula which they could use for the algebraic determination of one root of the cubic equation when it has one positive and two imaginary roots.

The Italian mathematicians began about the same time a much more fruitful arithmetization by the occasional use of complex numbers. This use was suggested, in particular, by the fact that when a cubic equation has three positive roots their formula, due to Ferro (1465–1526), but commonly known as *Cardan's formula*, presents these roots in the form of complex numbers. In fact, this is always the case when the three roots are real numbers, but negative roots were usually excluded by the mathematicians of the sixteenth century, and hence the fact that they also appeared in the form of complex numbers did not arouse much interest at that time. Even the noted English mathematician T. Harriot (1560–1621) thought he could prove that an algebraic equation could have only positive roots. Cf. *Artis analyticae praxis*, page 89. This is the more noteworthy because Harriot was not only a renowned mathematician but he was also prominently connected with the early settlement of our country, having surveyed and mapped the region now known as North Carolina.

Even the theory of the ordinary complex numbers, which were used tentatively from about the middle of the sixteenth century until the close of the eighteenth century when they were placed on a firm basis by Caspar Wessel and others, was arithmetized about thirty years later by means of a theory of number couples due to the noted Irish mathematician W. R. Hamilton (1805–1865). This arithmetic theory was greatly extended

later by means of the modular systems due to L. Kronecker, who emphasized the arithmetic foundation of the entire field of mathematics by the following words: "God made the integers, all else is the work of man." Cf. *Juhresbericht der deutschen Mathematiker-Vereinigung*, volume 2, 1893, page 19. The extent to which algebra had been arithmetized by the close of the nineteenth century is clearly exhibited in the standard work entitled *Lehrbuch der Algebra* by H. Weber.

The arithmetization of trigonometry is most clearly seen in the change as regards the definitions of the elementary trigonometric functions. With possibly a few isolated exceptions these functions were regarded as line segments up to the time of L. Euler (1707–1783). What is more important is the fact that they were not usually regarded up to this time as functions of a single variable, viz., the magnitude of the angle, but as functions of two variables, composed of this magnitude and the length of the radius of the circle or the hypotenuse of the triangle with respect to which the angle was considered. In fact, the main advantage which the arithmetization of the definitions of the elementary trigonometric functions entailed was due to the fact that these functions thereby became functions of a single variable. It may be added that even Euler did not define the elementary trigonometric functions as abstract numbers but he obviously used them with this meaning and this usage was soon followed by our modern definitions. Such definitions began to appear before the death of Euler but they were not firmly established until after the beginning of the nineteenth century.

It is interesting to note that the definitions of the elementary trigonometric functions as abstract numbers entailed the use of negative numbers when angles larger than 90° were considered. The imperfect knowledge as regards negative numbers even in the eighteenth century doubtless retarded the arithmetization of the definitions of the elementary trigonometric functions, and the usefulness of these numbers for this arithmetization helped to secure for them a permanent and prominent position among the elements of our subject. Even before the use of negative numbers was firmly established in elementary trigonometry the complex numbers began to enter the more advanced parts thereof. In particular, in 1740, Euler communicated the elegant formula $2 \cos x = e^{ix} + e^{-ix}$ to John Bernoulli. There was no generally accepted theory of the elementary operations with negative numbers until the more general subject of operating with the ordinary complex numbers had been placed on a firm basis.

The history of the arithmetization of analytic geometry is, as might have been expected, much more extensive than that of the arithmetization of trigonometry. It is frequently said that systems of coördinates are ancient, but these ancient systems had to be arithmetized. What is now generally understood by a system of coördinates, i.e., a system in which pairs of

real numbers with their proper signs represent points in the plane, and triplets of such numbers represent points in space, is comparatively modern, having been slowly introduced after the time of R. Descartes (1596–1650). "Descartes, Fermat and their successors regarded like the ancients their coördinates as line segments, which had a geometric connection with the separate points of the curves under consideration. Only since Euler, Lagrange and Monge does the purely analytic character of the chosen coördinates present itself more clearly." J. Tropfke, *Geschichte der Elementar-Mathematik*, volume 6, 1924, page 122.

It should be noted that while the arithmetization of systems of coordinates entailed the equivalence of the coördinates as well as the use of negative coördinates, this equivalence and this use were not dependent upon the arithmetical character of the systems of coördinates. In fact, negative coördinates were employed correctly as early as 1656 by J. Wallis, and the equivalence of the four quadrants was implied in I. Newton's *Enumeratio linearum tertii ordinis*, 1704, but many later writers did not adopt these advanced views. It is a matter of fundamental importance in the history of mathematics to determine when certain modern views became so well known that practically all of the best writers on the subject adopted them. Earlier isolated cases of such views are also of great interest, and instances where later writers deviated from these views are often instructive, but such isolated cases are usually of secondary importance. Barring such cases it may be said that the arithmetization of coördinate systems was effected about the same time as the arithmetization of the definitions of the elementary trigonometric functions; i.e., about the close of the eighteenth century.

The arithmetizations to which we have referred relate to fundamental subjects and hence they imply arithmetizations in the more advanced subjects based thereon. The main object of the present note is to direct attention to a unifying principle in the history of mathematics, and to cite a sufficient number of instances where it may be used to advantage to exhibit its value even in the study of the history of elementary mathematics. Incidentally the importance of accurate definitions of some of the terms used in historical discussions is also illustrated, for it is practically futile to state when systems of coördinates were first used without noting at the same time what is to be understood by such a system. Similar remarks apply to many other historical questions relating to the development of mathematics. Without such explanations the technical terms should be used only with their generally accepted modern meanings.

SKIN TEMPERATURE AND HEAT LOSS

BY FRANCIS G. BENEDICT

NUTRITION LABORATORY OF THE CARNEGIE INSTITUTION OF WASHINGTON, BOSTON, MASSACHUSETTS

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The body temperature is usually taken either in the mouth or rectum or, in the case of infants, in the axilla or groin. The normal body temperature is generally considered to be 98.6°F., but there are fluctuations from this average which, with the ordinary individual, may amount to 2°F. in 24 hours, the minimum temperature occurring at about 4 A.M. and the maximum about 5 P.M. under conditions of repose, with higher temperatures during severe muscular work. As an indication of the presence or absence of fever, the temperature taken in any of the localities cited is perfectly satisfactory, but as a means of studying the total heat stored in the body such temperature records are very deceptive. The temperature in the mouth, axilla or groin, for instance, is almost invariably somewhat lower than that in the rectum. Thus, a gradient exists between the temperature deep in the trunk and the temperature near the surface of the body. This gradient has been studied and it has been found that between the temperature deep in the trunk and that 1 cm. from the orifice of the rectum or vagina there is a difference of from 2° to 3°C.[1]

The regulation of the body temperature through the circulation of the blood and through loss of heat is usually so perfect that a lowered temperature in one part of the body due to the diurnal change, for example, is paralleled in the other parts of the body. The temperature of the exterior of the body, the skin, is so influenced by the environmental temperature that it is subject to much greater fluctuations. With humans, parts of the skin are exposed continuously, such as on the forehead, face and hands, and in the case of women the neck and not infrequently the arms and shoulders are exposed. Parts of the body are well covered, especially the abdomen and the small of the back, while other parts are but scantily covered, such as the arms and legs. With men the protective clothing is more evenly distributed. Save in summer weather the skin is always warmer than the air, and consequently heat is lost to the environment. Is this exactly and fully compensated? Does the skin temperature stay at or near the body temperature? Does it remain constant? Is the source of the heat supply so rapid and continuous as to compensate exactly for rapid and continuous changes in environmental temperature?

Measurements of the skin temperature are now necessary in estimating and particularly in interpreting the loss of heat to the air. Unfortunately this temperature is measured only with difficulty, and mercury thermom-

eters are of little use. The only satisfactory method is that of the thermojunction. The idea of this apparatus is very old, but with its modern application it is possible to make a rapid survey of the skin temperature with an accuracy of 0.1°C. Indeed, by photographing the movements of a string galvanometer extraordinarily sensitive records may be obtained.[2] By means of this apparatus many surveys of the skin temperature have been made at the Nutrition Laboratory, to study the effect of varying amounts of protective clothing and changes in environmental temperature. In most of these surveys the rectal temperature was not recorded, but there are a sufficient number of records to justify the statement that the rectal temperature was not far from 37°C., plus or minus 0.5°C.

Numerous conditions were included in this study of the skin temperature. Thus, a survey of the skin temperature was made (*a*) after a night's rest in bed, when the subjects were well covered with bed clothing; (*b*) under the clothing, after the subjects had been lying for 2 or 3 hours in a known environmental temperature; (*c*) under the clothing with the subjects standing, after 4 to 6 hours in the temperature of the laboratory; and (*d*) under the clothing after prior exposure to external weather conditions. This latter survey led to a study of the immediate effect upon skin temperature of exposure to cold. Finally, the relation of skin temperature to heat loss has been considered from the standpoint of the direct radiation of heat, as measured with a pyranometer, and an analysis of the total loss of heat in a calorimeter chamber, as divided between loss due to the vaporization of water and loss due to the direct radiation of heat.

A survey of the skin temperature of 12 girls, 14 years of age, was taken under the bed clothing, after a night's sojourn in bed with the body well covered. The measurements were made at approximately 25 points on the body, of which 3 were on the head and face, 8 on the ventral side of the trunk, 6 on the arms and hands, and 8 on the thighs, calves and feet. The average skin temperature of the 12 girls was 33.2°C., ranging from 32.9°C. to 34.1°C. There was in general a fairly equable temperature about the abdomen and trunk, with a tendency for the extremities to be somewhat cooler as the distance from the trunk increased.

Under the protective covering of the usual clothing and after 2 or 3 hours in the same environmental temperature each individual has provided for himself under the clothing a layer of warm air, which Dorno[3] has designated as his "private climate."

A survey taken when the subject had been lying, well covered for 3 hours, showed usually somewhat lower skin temperatures over the whole body than the temperatures noted after a night's repose in bed. The temperature of the extremities is greatly affected by the environmental temperature in the bedroom or that existing prior to the observation, the

placing of a hot-water bottle at the foot of the bed resulting in a very rapid and marked equalization of temperature over the entire surface of the body. Laboratory assistants, clothed and standing after 4 to 8 hours in the laboratory, showed much the same picture, with, however, a noticeable reaction to the environmental temperature, for on days in midsummer, with temperatures of about 30°C., the entire skin was at a much more even temperature than under any other conditions.

A temperature survey taken under the clothing, immediately after the subject entered the laboratory in the morning, showed invariably a distinct correlation between the skin temperature and the temperature of the exterior, this correlation being noted even in the well-protected parts of the trunk. Thus, on a very cold February day the entire trunk may have a skin temperature as low as 31°C., with the temperature of the legs and feet falling off to not far from 24° to 25°C.

With one subject rapid exposure to a cold environment was made by suddenly removing the clothing. The equable skin temperature existing in the "private climate" is almost instantly destroyed by exposure for but one or two minutes to a temperature of about 15°C.

The effect of muscular work upon skin temperature was pronounced in the case of a nude male subject. In this case the temperature of the right side of the body was measured at 13 points after one hour's exposure to a room temperature of 20°C. The average skin temperature of these points, which prior to exposure must have been not far from 33°C., fell with this subject to 30°C. The subject then began riding a stationary bicycle, and at the end of 13 minutes the average skin temperature had further fallen to 28.2°C., but as the work continued throughout the morning for over $2^1/_2$ hours, the average temperature did not go below 27.5°C., since the muscular work maintained the temperature at this level. Furthermore, the skin temperature of the legs, where most of the physical exercise was being carried out, in a number of instances actually rose as a result of the muscular work.

The significance of skin temperature in the loss of heat by radiation has been too long neglected. Thanks to the kind coöperation of Dr. Charles G. Abbot of the Smithsonian Institution, a series of observations was made by him at the Nutrition Laboratory upon one of our subjects, using his pyranometer, and it was found possible from his observations to predict closely the skin temperature. Thus, Dr. Abbot's observations point clearly to the fact that the human skin acts as a true "black body," and hence the temperature of the skin may be taken in conjunction with the temperature of the environment as an excellent indication of the loss of heat by radiation. Such a computation is complicated, however, in the case of the clothed individual, owing to the stagnant air of the "private climate" and owing to the fact that normally an appreciable proportion

of the heat is lost by the vaporization of water through the lungs and skin. Indeed, approximately one-third to one-quarter of the water lost from the lungs and skin (more in extremely warm weather), is that vaporized from the skin.

From calorimetric observations it has been found that the maximum amount of heat thus far measured as given off by direct radiation was in the case of a subject who rode nearly 20 hours on a bicycle ergometer, producing the total amount of 7137 calories per 24 hours, of which essentially one-half was lost by the vaporization of water vapor while 3656 calories were lost by direct radiation.

The significance of heat loss to the environment, its relation to environmental temperature, air movement, humidity and protective clothing, is only equalled in importance by a careful study of the heat production. Our observations lead us to the conviction that, save under extreme conditions, heat production and heat loss are two essentially independent processes. Very considerable fluctuations in the heat stored in or lost from the body are possible, resulting in rather profound changes in the temperature of the peripheral tissues without appreciable alteration in heat production.

The details of the temperature measurements are about to be given in a forthcoming number of Asher-Spiro's Ergebnisse der Physiologie.

[1] Benedict, F. G., and E. P. Slack, *Carnegie Inst. Wash. Pub.*, No. **155**, 1911.

[2] Benedict, F. G., W. R. Miles, and A. Johnson, *Proc. Nat. Acad. Sci.*, **5**, 1919 (218–222).

[3] Dorno, C., *Medical Climatology and High Altitude Climate*, Vieweg and Son, Brunswick, 1924 (58).

THE PASSAGE OF SLOW CANAL RAYS THROUGH HYDROGEN

By A. J. Dempster

Ryerson Laboratory, University of Chicago

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One of the well known properties of canal rays is that of changing their charge while passing through a gas. The charged atoms alter from positive to neutral or back again to positive at almost every collision with a gas molecule. Rückhardt[1] has investigated canal rays of positive hydrogen atoms having a velocity between 1.6×10^8 and 2.3×10^8 centimeters per second (13,000 to 26,000 volts), and found that the radius of the hydrogen molecule deduced from the number of collisions at which charge alterations occur varies from 1.13×10^{-8} cms. for the fastest rays to 2.0×10^{-8} cms. for the slowest.

Using methods that have been developed in the study of the free path of electrons, Aich[2] has found that very slow hydrogen nuclei with a velocity of 7×10^6 cms. per sec. (25 volts), apparently do not penetrate into the hydrogen molecule, but are scattered out of the beam; the radius of the molecule required to account for the scattering or absorption was found to be in fair agreement with the value deduced from the kinetic theory.

Canal rays of an intermediate velocity corresponding to a potential difference of 50 to 2000 volts have been investigated by the author.[3,4] It was observed that the constituents in the rays responsible for light excitation become neutralized as soon as a small amount of hydrogen is introduced. There were also indications that some of the neutral particles again become positive.

In the present experiments rays of positively charged hydrogen atoms or protons that have fallen through 900 volts were used. After being accelerated the rays passed through a slit and were bent into a semicircular path 15.7 cms. in length by a magnetic field and fell on a second slit, as described in previous papers.[5,6] The magnetic field was kept constant, and the accelerating voltage required to bring the rays on to the second slit when no gas was present was compared with the voltage required when hydrogen was present at various pressures. Two effects might be expected. With a pressure of 0.008 mm. of mercury the free path for neutralization of the slowest positive rays observed by Rückhardt would be 2.5 cms. Our rays had a considerably slower velocity of 0.416×10^7 cms. per sec., and his observations indicated a decrease in the free path with a decrease in the velocity. The free path for the change back to positive is 28 centimeters, so that in our experiment the rays may change back and forth from neutral to positive several times in describing the path between the slits. During the time that the atom is neutral, no magnetic deflection would occur, so that when hydrogen is present the rays should require a much stronger magnetic field or much slower initial velocity in order that they should fall on the second slit. An effect of the opposite kind might be expected if the protons should lose velocity in their passage through the gas molecules.

With a constant magnetic field the voltage was observed, in rapid succession, with hydrogen at pressures between 0.008 mm. and 0.00017 mm. of mercury. No alteration at all could be observed in the voltage and magnetic field required to keep the maximum of the rays on the second slit, although a change of 2 volts in the 900 applied could have been detected. We must conclude then that neither of the effects mentioned above has an appreciable effect on the majority of the particles in the beam. The two effects could not exactly compensate over a range of pressures, since the dependance on pressure is different in each case. It is not possible to consider the rays observed with gas present to be the small

fraction that have traversed the distance between the slits without undergoing a collision, since with the value of the mean free path given above this fraction is of the order of 10^{-3}, and the intensity of the rays was found to be about the same for the same anode temperature whether gas was present or not.

The conclusion that for the majority of the rays no loss of energy occurs as great as that corresponding to a fall through two volts potential difference indicates that the proton of this speed will pass through a great many molecules and not ionize or excite light, since each of the processes requires more energy than was lost by the protons. The absence of neutralization finds an interpretation on the same basis. The protons pass through the molecules without detaching an electron with which they could combine. There probably exists a critical velocity between 900 and 13,000 volts at which protons acquire this ability to excite or ionize hydrogen, and also a lower critical velocity between 25 and 900 volts, at which the proton loses its ability to pass through the molecules.

Since the presence or absence of an effect of a proton of any velocity on a molecule must depend on the electric forces involved and their variation, rather than on any influence in which the mass of the particle would be important, we should be led to expect analogies in the behavior of protons and electrons of the same velocity. Electrons acquire the velocity of the protons used in these experiments by falling through 0.5 volt, and it is significant that electrons of this speed possess the ability, in the case of several gases, of passing through many molecules without absorption.

Summary.—Protons with a velocity of 4.16×10^7 centimeters per second (900 volts) will pass through many hydrogen molecules without being neutralized. Their velocity is not altered by as much as two volts in passing through the molecules.

[1] Rückhardt, E., *Ann. Physik*, **73**, 1924 (230–236).
[2] Aich, W., *Z. Physik*, **9**, 1922 (372–378).
[3] Dempster, A. J., *Proc. Nat. Acad. Sci.*, **2**, 1916 (374–376).
[4] Dempster, A. J., *Physic. Rev.*, **8**, 1916 (651–662).
[5] Dempster, A. J., *Ibid.*, **11**, 1918 (316–325).
[6] Dempster, A. J., *Proc. Nat. Acad. Sci.*, **7**, 1921 (45–47).

ENERGY TRANSFORMATIONS IN AN UNOBSTRUCTED AIR CURRENT, IN AN AIR CURRENT CONTAINING A DRY OBSTACLE, AND IN AN AIR CURRENT CONTAINING AN EVAPORATING SURFACE; WITH APPLICATIONS TO AN AERODYNAMIC PSYCHROMETER AND TO THE MEASUREMENT OF ATMOSPHERIC HUMIDITY

BY GEORGE PORTER PAINE[1]

JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY

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1. *Introduction.*—The wet bulb hygrometer, or psychrometer, as it has been called since the appearance of August's paper on that instrument in 1825,[2] presents a problem in the transfer of energy in a flowing gas mixture which is of interest in all of its aspects, theoretical and experimental. Clerk Maxwell[3] and Stefan[4] endeavored to explain the action of this instrument in the light of the Kinetic Theory of Gases; Regnault[5] subjected it to an elaborate experimental investigation, and a large amount of work has been done on the psychrometer during the eighty years that have elapsed since Regnault's time.[6]

Singularly enough, little attention has been given to effects on the behavior of this instrument of the following forms of energy manifested by the moist air moving past the thermometer bulbs, viz.: (*a*) heat energy conducted to (or from) the moist air, (*b*) energy dissipated by viscous stresses brought into existence by space rates of change of deformation velocities of the moist air, (*c*) heat energy of turbulent motion dissipated by the moist air, and (*d*) translatory work of the moist air against mean static pressure.

Transformations of these forms of energy of the moist air near the wet bulb are by no means necessarily identical with those near the dry bulb, and the differences should be taken into account in calculations based on readings of the psychrometer.

General considerations of this nature led the writer to undertake an investigation of turbulent air flow and evaporation in a wind channel designed for the purpose, and subsequently to extend the research to a psychrometer designed to admit measurement of the velocity of the air.

For an account of this investigation, the reader is referred to the *Annals of the Astronomical Observatory of Harvard College*, 1925, **87**, No. 1, papers I to V inclusive. Some of the results there set forth are briefly outlined in the following pages.

2. *Analysis of Energy of Air Flow.*—Assuming that the internal stress components of the moist air flowing past the thermometer bulbs are linear

functions of the strain velocities, the differential equations for the moist air under consideration may be obtained by writing the standard hydrodynamic equations for a viscous, compressible fluid, and then adding the terms derived by Maxwell for the forces of diffusion. These equations, as is well known, can be so combined as to yield an energy relation between non-dissipative forms of the energy of the air. A second relation is obtained by writing the Law of Conservation of Energy. By combining these two relations, a relation between the rate of change of the heat energy, and that of the dissipative forms of mechanical energy of the gas mixture can be deduced. This third relation, and the first of the relations above mentioned, together with the equations of continuity and the characteristic equations of the media, constitute the theoretical basis of the investigation.

All of the experiments referred to below were performed with an air flow exceeding the critical velocities of the apparatus. But the *detailed* motions and changes of state described by the classical equations are not observable in the case of turbulent media by means of ordinary aerodynamical instruments, which render mechanically averaged mean values of these detailed motions and changes of state. The Reynolds-Lorentz[7] mean value analysis was accordingly applied to the first and the third of the above-mentioned energy relations. Two fundamental mean value equations were thus obtained. The first of these consists of terms in the mean motion, and a term Ψ representing energy due to the turbulent motion of the air. The latter arises from (*a*) turbulent changes of kinetic energy of the fluid, (*b*) translation work performed by the air and due to turbulent changes of static pressure, (*c*) work performed by the body forces through turbulent vertical displacements of the fluid elements and (*d*) the non-dissipative work of internal friction brought about by turbulent changes of the deformation velocities of the medium. On account of its non-dissipative character, this function Ψ may be regarded as defining the *mechanical energy of turbulent motion*. Measurements made in uniform portions of air currents used by the writer show that, up to 31.4 meters per second, the algebraic sum of the mechanical energy of turbulent motion, and that part of the non-dissipative work of the viscous stresses which was due to the mean motion, was zero, to a precision within the accuracy of the Pitot tube. The mean flow of the portions of air currents under consideration accordingly obeyed approximately Bernoulli's Law.

Mean value analysis, applied to the energy relation containing dissipative terms, led in an analogous manner to an equation containing mean motion terms and a function $(-\Phi)$ which, from its dissipative character, has been referred to by the writer as the *heat energy of turbulent motion*.

Taking advantage of certain characteristics of the mean value terms which are due to the design of the apparatus, simple approximations can

be made which render it possible partly to integrate the differential equations and thus calculate effects of viscosity, heat conduction and turbulence, in terms of the observable variables of the mean motion. These mean motion variables were ascertained by direct measurements.

3. *Experimental Results in an Unobstructed Channel.*—In order to eliminate as far as possible effects of heat radiation, and to obtain a uniform air jet, the writer used a small, burnished brass replica of M. Eiffel's wind channel. Air was drawn through this small channel by an exhauster at velocities up to 31.4 meters per second. The jet thus produced was characterized by uniform *mean* static and dynamic pressures over a central cross-sectional area of about 7 centimeters in diameter. Air temperatures were measured by means of sets of minute, high-precision copper-constantan thermo-couples, which were effectively shielded from radiation effects by suitable surfaces of polished metal.

It was found that air, entering the channel at room temperature, undergoes a distinct cooling by the time it reaches the experimental cross-section of the channel; and this cooling effect bears a definite relation to mean wind velocity. Experiments show that the cooling effect under consideration is due to the performance of dilatation work by the air against mean static pressure and at the expense of heat loss corresponding to a fall of mean air temperature and at the expense also of forms of energy denoted by the symbol E in the papers by the writer above referred to. This function E is the algebraic sum of (1) gain of heat energy of turbulent motion, (2) heat conducted into the moving air mass by virtue of space variations of mean air temperature and (3) energy dissipated by the internal stresses of the air in raising mean air temperature and in performing dilatation work against mean static pressure.

Of appreciable relative magnitude, this function, under standard atmospheric conditions, obeys the empirical law:

$$E = 6.89 \times 10^{-4} \times \bar{q}^{1.773} \qquad (1)$$

where E is in joules per gram of air, and q is mean flow velocity, in meters per second.

These results follow from the third (dissipative) energy relation referred to above. Experiments in conjunction with the first (non-dissipative) energy relation show that the uniform portion of the jet under consideration obeyed Bernoulli's law.

4. *Experimental Results in a Channel Containing a Dry Obstacle.*—To study energy transformations in the wake of an obstacle, the writer set up a burnished brass rod in the channel referred to in the last section, and he measured pressures, temperatures and velocities in its wake. This investigation is described in the second of the papers in the *Annals*. Re-

sults were made possible, as in the case of the unobstructed channel, by the use of thermoelectric methods of temperature measurement and by effective screening of the thermojunctions from effects of radiation. When mean flow velocity is increased from zero up to about 4 meters per second, eddy temperature[8] increases to a maximum. When flow velocity is further increased, eddy temperature undergoes a systematic decrease from zero, at a velocity of 12 meters per second, to (−0.17°C.) at a velocity of 33.5 meters per second. The warming effect at low velocities probably corresponds to a stream-line *régime* between velocities of 0 and about 0.37 meter per second, and, between velocities of 0.37 and about 2 meters per second, to a *régime* of channel turbulence preceding the appearance of the eddy in the wake of the obstacle.

Eddy pressures[9] increased systematically with velocity.

Measurements show that the energy denoted by the integral E, defined above in section 3, is dissipated, when the air is moving with low velocity, in increasing mean air temperature and in performing dilatation work against mean static pressure. For higher velocities, dilatation work against mean static pressure is performed by the air, not only at the expense of the heat energy of turbulent motion, etc., denoted by E, but also at the expense of the internal heat energy of the air. The algebraic sum of energy transformations represented by the integral E reached a maximum at a mean flow velocity of about 4.2 meters per second, and dropped to a minimum at about 18.3 meters per second; from which minimum it increased with mean flow velocity throughout the remainder of the range of velocities investigated.

5. *Wick Characteristics.*—In order effectively to study energy transformations in a jet of air flowing past a wet obstacle, it is obviously essential to produce and to maintain experimentally determinate boundary conditions at the wet surface. With this end in view, the writer covered one of the small thermo-couple bulbs with a wick saturated with distilled water. Wicks of different types and materials were used, and a variety of means employed for keeping the wick saturated with water. Characteristic curves were obtained by reading the wick temperature at short intervals, and then plotting the result as a function of the time.

During these tests, experimentally determinate boundary conditions could not be maintained when the wick was moistened by means of a continuous feed. Apparently, therefore, the psychrometer (unless very greatly differentiated from any types now known) cannot properly be used as a continuously self-recording instrument. Wet bulb depressions of an instrument of this type are seriously affected by states of equilibrium between rate of evaporation and rate of feed to the wick; and the latter appears inevitably to depend upon conditions which are experimentally indeterminate. The reader will find wick characteristic curves, together

with a discussion of conditions referred to, in the third of the papers in the Harvard *Annals*.

Conditions on the wet boundary are, however, experimentally determinate under the following restrictions:

(1) The wet bulb must be thermally insulated from its surroundings.

(2) It must be effectively shielded from radiation exchanges with neighboring material at a different temperature.

(3) The wick must consist of bulb covering only, without continuous feed of any kind, and it must fit tightly, without wrinkles.

(4) Complete saturation of the wick must be effected by immersion in distilled water immediately before exposure to the air current.

(5) The material, shape and texture of the wick must be such as to maintain exposed to the air a *continuous* film of liquid on the enclosed glass tube for an appreciable length of time.

(6) Mean wind velocity over the wet surface should lie within the range between 3 and 15 meters per second. Air having a velocity exceeding 15 meters per second carries spray from the wet bulb, thus giving rise to conditions not contemplated in the present investigation.

When the foregoing restrictions are complied with, the mean wet bulb temperature falls to a minimum M immediately after it has been exposed to a current of air, and it remains at this minimum for a time sufficiently long to define a steady mean state. The difference between the temperature of the dry bulb and this minimum temperature M depends upon atmospheric temperature, pressure and humidity, and also upon the configuration of the solid boundaries of the apparatus,[10] and upon these variables alone.

6. *Energy Transformations at the Wet Bulb and at the Dry Bulb in a Turbulent Current of Moist Air.*—Utilizing the same wind channel as before, measurements were made of energy transformations taking place in the air, from the time it started on its course into the channel, until it reached the center of intensity of the wet bulb eddy. The wet bulb consisted of a minute thermo-couple set immersed in light oil and the whole contained in a small-diameter, thin walled, glass tube, and covered with a thin, smoothly fitting, chemically clean wick.

Differential equations, set up, as before, for the air current, show that the work done by forces of diffusion cannot be dissipated in heat nor in causing the moist air to perform dilatation work against mean static pressure.

Making use again of simple approximations permitted by the character of the phenomena under consideration, these equations can be partly integrated along any stream-line between two points at which boundary conditions can be ascertained. The solutions contain an energy integral representing the algebraic sum of (1) heat energy conducted to (or from)

the air mass under consideration by virtue of the space rates of change of mean air temperature, (2) energy dissipated by the internal stresses due to space rates of change of mean deformation velocities, (3) heat energy of turbulent motion of the air, and (4) translatory work of the air against mean static pressure.

Measurements show that this algebraic sum is greater when the integration is performed along a mean stream-line from still air to the wet bulb, than it is when the integration is performed along a mean stream-line from still air to the dry bulb.

This excess E persists when atmospheric conditions prevent evaporation from the bulb covering, whence it is to be inferred that the comparatively rough surface of the bulb covering intensifies turbulent motion, and increases space rates of change of mean deformation velocities of the air at the wet bulb, thus increasing there the energy dissipated by the viscous stresses in the air, and consequently augmenting the algebraic sum under consideration.

Precision measurements made under a variety of atmospheric conditions show that this excess E is (within certain limits discussed below) independent of mean flow velocity and atmospheric conditions; it is, in fact, an instrumental constant.

7. *Dynamical Cooling without Evaporation. A Necessary Condition Restricting the Effect Range of Wet Bulb Depression.*—Dynamical cooling, described above in section 3, gives rise, in general, to part of the wet bulb depression. When the water vapor in the air is sufficiently near saturation, the entire wet bulb depression is due to this effect. The relation between wet bulb depression and atmospheric humidity is accordingly[11] experimentally determinate only when wet bulb depression satisfies the following necessary condition, viz.:

$$(t-t') \geqq \left(\frac{E}{1.011}\right)\left[\frac{1 + 0.621\left(\frac{P'}{b-P'}\right)}{1 + 1.162\left(\frac{P'}{b-P'}\right)}\right] \qquad (2)$$

where t = dry bulb temperature, t' = wet bulb temperature, P' = pressure of saturated water vapor at wet bulb temperature t', b = barometric pressure, E = an energy integral shown by the writer to be an instrumental constant.

Temperatures are measured on the centigrade scale, and pressures are measured in millimeters of mercury. E is in joules per gram of air.

8. *Psychrometric Formula Derived from the Foregoing Considerations.*—The (dissipative) energy relation forming the basis of the foregoing results

leads to the psychrometric formula

$$\begin{aligned} p &= p_1 + Q \\ p_1 &= P' - \frac{0.3891\left(\frac{t-t'}{L'}\right)}{1 + 0.453\left(\frac{t-t'}{L'}\right)} \\ Q &= (6.55 \times 10^{-4}\, E)\,(b - P') \end{aligned} \tag{3}$$

where p = pressure of water vapor in the air, L' = heat of vaporization of water at wet bulb temperature, in calories per gram. (For units and the remainder of the symbols, see foot-note to section 7.)

9. *The Aerodynamic Psychrometer. Precision of Instrument and Formula.*—In order to realize more fully the conditions under which the foregoing formula was derived, and to render the apparatus portable, a psychrometer was constructed and tested under a variety of atmospheric conditions. The thermometers of this instrument are mounted on a frame similar to that of the well known Assmann instrument. Double Assmann shield tubes, silver plated and burnished, were used to screen the bulbs from heat radiation, these tubes being relatively somewhat larger than those used by the German Meteorologist.

A longitudinal partition inserted in the central channel tube extended down to the junction of the thermometer tubes, in such a manner as to prevent mixing of the cool air from the wet bulb shield tube with the warm air from the dry bulb shield tube. Without this partition, cool air is propagated backward into the dry bulb shield tube by the eddy formed at the junction of the two thermometer tubes, thus causing a depression of the dry bulb. This effect was noticeable for forced drafts above 3 meters per second.

Air is sucked into the shield tubes by means of an exhauster provided with a 12-blade rotor and driven by a small electric motor. Forced draft in the shield tubes containing the thermometer bulbs is adjusted by means of a relief valve in the central channel tube. This adjustment can be made with great precision with the aid of a water manometer leading from the channel tube. In order to calibrate this manometer for forced draft on the thermometer bulbs, the rotary fan exhausts into a long 2-inch calibrating tube, in which is installed a Pitot tube leading to a Krell gage. From the measured discharge rate of the calibrating tube, mean flow velocity over the thermometer bulbs is readily calculated.

In order experimentally to verify the psychrometric formula given in the last section, a chemical hygrometer was installed at one side of the psychrometer, with its intake nozzle 15.8 centimeters below the entrance nozzles of the psychrometer, and in the axis of symmetry of the latter

instrument. Wicks used for this psychrometer are made of the finest linen lawn and fit the thermometer bulb perfectly without trace of looseness or wrinkles. Immediately before each observation, the wick is completely saturated by inserting a glass tube of distilled water up into the shield tube, thus immersing the wet bulb.

Under these conditions, a wet bulb minimum M, of the type affording well-defined, steady mean value boundary conditions as developed in the third paper of the *Annals* series, persists over a period amply sufficient for experimental purposes, provided the diameter of the cylindrical wet bulb is near 6.5 millimeters. Decreased wet bulb diameter results in increased evaporation rate, and a consequent shorter duration of the required wet bulb minimum M.

For this instrument, the constant E was found to be 0.727 joule per gram of atmospheric air. (For the thermo-couple bulbs in the wind channel, the value of this instrumental constant was found to be 0.84.)

Tests with the aid of the chemical hygrometer indicated an agreement between the aerodynamic psychrometer and the chemical hygrometer within 1.3 per cent in the case of absolute humidity, and within 0.8 per cent in the case of relative humidity, thus establishing the practial validity of the formula for the aerodynamic psychrometer.

[1] NATIONAL RESEARCH FELLOW, 1919–1923.

[2] *Ann. Physik. Chem.*, **5**, 71 (1825).

[3] *Encyclopedia Britanica*, Ninth Edition, Edinburgh, Vol. VII, 218 (1877). (Article on *Diffusion*.)

[4] *Zeitschr. Österreich. Gllsch. Met.*, **16**, 177 (1881).

[5] *Ann. Chim. Phys.* (3) **15**, 129 (1845), and (3) **37**, 257 (1853).

[6] See Paine, *Annals of the Astronomical Observatory of Harvard College*, **87**, No. 1 (1925), fifth paper, Appendix.

[7] Reynolds, *Lond. Roy. Soc. Phil. Trans.*, **186**, 123 (1865); Lorentz, *Abhandlungen über Theoretische Physik*, Leipzig u. Berlin, 1907, third memoir, p. 43.

[8] Air temperature in the eddy in the rear of the obstacle, minus air temperature at a point sufficiently in advance of the obstacle not to be affected by the latter's presence.

[9] Static pressure sufficiently in advance of the obstacle not to be affected by its presence, minus statis pressure in the eddy.

[10] The latter property of the wet bulb depression is experimentally demonstrated in the fifth paper of the *Annals* series.

[11] See third paper of the *Annals* series, section 27 on *Dynamical cooling without evaporation*.

NOTE ON THE EXPERIMENTAL DETERMINATION OF THE RELATIVE INTENSITIES OF SOME OF THE MOLYBDENUM AND COPPER K SERIES LINES AND THE TUNGSTEN L SERIES LINES

BY SAMUEL K. ALLISON[1] AND ALICE H. ARMSTRONG

CRUFT LABORATORY, HARVARD UNIVERSITY

Communicated July 16, 1925

The first precision measurements of the relative intensities of X-ray lines were those of Duane and Stenström[2] on the K series of tungsten and Duane and Patterson[3] on the L series of tungsten and the K series of molybdenum. The relative intensities of the K series lines of rhodium are given in curves shown still earlier by Duane and Hu[4] in their study of absorption and emission frequencies.

Further interest in the experimental determination of the relative intensities of X-ray lines has been stimulated by the recent extension[5] to X-ray spectra of the rules which Burger and Dorgelo[6] have given for relative intensities in the optical region.

We have recently redetermined the relative intensities of the strong L series lines of tungsten and the K series lines of molybdenum, and have also studied some of the copper K series lines and the weaker lines in the tungsten L spectrum. Our investigations were made by means of the ionization method used by Duane and his collaborators. Our apparatus, however, had several improvements over that with which their work was performed. The crystal table could be turned with precision through a few seconds of arc, and its position read on the Génévoise scale with which the spectrometer was provided. The employment of tubes with water-cooled targets permitted the use of much more power than that used in the previous experiments. The source of power was a new high-capacity storage battery capable of furnishing 50,000 volts and allowing operation of the tube at high power for much longer intervals than before.

For the experiments on the molybdenum spectrum we used tubes with water-cooled molybdenum targets. For the study of the tungsten and copper spectra we had a tube with a water-cooled anode which consisted of a tungsten button embedded in copper. The copper K lines were emitted strongly from the copper part of the target. The radiation used in the spectrometer emerged from the tube through a very thin mica window on the end of a long arm.

Except in the investigation of the molybdenum Kβ doublet, the reflected beam of X-rays entered the ionization chamber through a very thin mica window, as in the experiments of Duane and Patterson on the tungsten L series lines. The ionization chamber was filled with methyl iodide vapor.

The K Series Lines.—The ratio 2:1 predicted by the extension of the rules of Burger and Dorgelo for the components of the Kα doublet has been verified for a wide range of elements by the researches of Duane and Stenström,[2] Duane and Patterson,[3] and Siegbahn and Záček.[7] We have not made a careful study of these lines. An interesting interpretation of the relative intensities in the Kα doublet has been given by Stoner,[8] who has recently proposed a new scheme of electron distribution among atomic levels. He considers that the observed ratio 2:1 may be explained on the assumption that there are twice as many electrons in the L_{22} orbit as in the L_{21} orbit.

In our recent separation of the Mo Kβ doublet[9] in the 4th and 5th orders we found the relative intensity β_1: β_3 to be 2:1, in agreement with the predictions of Sommerfeld and Coster and Goudsmit. According to Stoner's scheme the observed ratio could be used as an indication that there are twice as many electrons in the M_{22} orbit as in the M_{21} orbit, provided the electrons in the intervening L orbits be assumed to have little disturbing influence on the transitions from the M group to the K group.

We have not been able to test the theoretical preductions for the Kγ doublet, as the two components have not yet been separated.

We have made several measurements of the relative intensity of Mo Kβ to Kγ in the first order, and have found for the average value 7.7:1. Duane and Patterson gave 6.3 for the first order and 5.46 for the second order. They obtained these ratios by assuming the intensity of each peak to be proportional to the height of the peak above the base-line. This method is sufficient for a first approximation; but, since both the β and γ lines are doublets of unequal wave-length separation, a better method is probably to take the area under each peak as proportional to the intensity. This we have done, and we have also applied corrections, in the case of the γ line, for the critical absorption due to the target. In these determinations the variation of the voltage and the tube current through a fairly wide range appeared to have no effect upon the relative intensities.

The average of our two determinations of the relative intensity of Cu Kβ to Kγ gives the ratio 100:2.4, or 42:1. Since, however, the γ line is very weak, and the critical absorption lies near it, estimates of its intensity are not very reliable.

Duane and Stenström found the relative intensity of β to γ in the K series spectrum of tungsten to be 35:15, or 2.3:1. The intensity of Kβ relative to Kγ for rhodium, obtained from the curves shown by Duane and Hu, lies between those for tungsten and molybdenum.

These values for the ratio Kβ: Kγ illustrate the decrease in intensity of γ with decreasing atomic number. No quantitative theory has yet been proposed to account for the relative intensities of such lines.

The intensity of Kβ relative to Kα has been measured for several ele-

ments by Unnewehr[10] and by Siegbahn and Záček.[7] On account of the large correction for absorption by the targets of our X-ray tubes, we have been unable to obtain satisfactory values for this ratio, although Unnewehr has applied to his measurements a correction for this absorption effect.

We have also investigated the copper spectrum in the region of the non-diagram lines $K\alpha_3$, $K\alpha_4$, reported by Dolejsek and Dolejsek and Siegbahn,[11] and interpreted as spark lines by Wentzel.[12] Although the Cu $K\alpha_1$ and $K\alpha_2$ lines were very strong, there was no satisfactory evidence of the $K\alpha_3$, α_4 lines at 30,700 volts. The intensity of this doublet relative to $K\alpha_1$ must be less than 1:100.

The L Series Lines of Tungsten.—We have made three determinations of the relative intensity of $L\alpha_1$ to $L\alpha_2$ and find for the average the value 10.0:1, the same as that obtained by Duane and Patterson. The ratio predicted from the extension of the rules of Burger and Dorgelo is 9:1. Sommerfeld[5] has pointed out that according to Stoner's scheme the ratio would be 6:4, and has remarked that the simple interpretation of relative intensities in terms of the relative numbers of electrons in the orbits is evidently inadequate in the case of compound doublets such as $L\alpha_1$, α_2, β_1.

In addition to the strong lines of the β group, β_1, β_2, β_3, β_4, we have measured also the lines β_5, β_6, β_7, β_9, β_{10}. We have not found evidence of the line β_8, and the lines β_{15} and β_{16} of Crofutt,[13] although our failure to do so is doubtless due to the fact that our slits were not sufficiently narrow. An attempt is now being made to separate β_2 and β_{15} in the third order.

Duane and Patterson found β_1, β_2, β_3, β_4 proportional to the numbers 100, 55, 16, 9. The average of our series of measurements of the β group gives the relative intensities of the lines β_1, β_2 ,β_3, β_4, β_5, β_6, β_7, β_9, β_{10} proportional to 100, 49.3, 15.0, 7.7, 0.47, 2.0, 0.4, 0.68, 0.60. Since β_9 and β_{10} are on the short wave-length side of the absorption L_{22}, which appeared as the critical absorption of the target, the values given for their intensities relative to the other β lines can be considered only as minimum values. These two lines appeared to be of about equal intensity. The L_{22} absorption also complicated the choice of a proper base-line for β_5. The line β_6 appeared as a shelf on the long wave-length side of β_1. The values of the relative intensities of these weak lines among themselves are thus not very reliable; but they serve, at least, to give an idea of the magnitude of the lines relative to the stronger ones.

According to our measurements the lines β_3, β_4 have a relative intensity which is close to the ratio 2:1 predicted by Sommerfeld and Coster and Goudsmit. Stoner has remarked that the ratio of β_3 to β_4 obtained by Duane and Patterson suggests the ratio 2:1 for the relative numbers of electrons in the M_{22} and M_{21} orbits.

We have also measured the relative intensities of the lines γ_1, γ_2, γ_3, γ_4, γ_5, γ_6. In some instances we obtained evidence of γ_8, but were not

always able to measure its intensity, as the height of the peak above the general radiation varied. The estimation of the intensity of γ_6 was complicated by the L_{21} absorption, which lies close to the line.

The results of our determinations give the intensities of the lines γ_1, γ_2, γ_3, γ_4, γ_5, γ_6, γ_8 proportional to the numbers 100, 14.0, 22.3, 7.0, 3.0, 2.3, 1 (?). Duane and Patterson found the first four of these lines proportional to 100, 14, 18, 6. If their curves are corrected for incomplete resolution by a method described elsewhere[14] by us, a value of the ratio $\gamma_3:\gamma_2$ is obtained which is nearer our value of 1.59:1. In this case, then, there is an apparent exception to the extension of the rules of Burger and Dorgelo and to Stoner's scheme, according to both of which the ratio should be 2:1. Since the lines are close together, corrections for absorption by materials in the path of the beam are negligible. On the basis of the measurements of Duane and Patterson, Stoner has suggested that the failure of the simple ratio to hold in this case, in which the transitions are from the N to the L groups, may be due to disturbing factors introduced by the intervening electrons of the M group.

The measurements of relative intensities such as $\alpha_1:\beta_1$ and $\beta_2:\gamma_1$, for which the ratio predicted by the extension of the rules of Burger and Dorgelo is 9:5, is of great interest. In our researches such determinations were complicated by the absorption in the target; consequently our results are not of great value in testing theoretical predictions. We found α_1 to be slightly less intense than β_1, and β_2 to be less than twice as intense as γ_1. The correction for absorption, however, could not bring both ratios to the value of 9:5, for it would change both in the same direction.

More detailed accounts of these investigations will appear soon.

We wish to acknowledge our indebtedness to Professor William Duane, who suggested this problem and has given much valuable criticism.

[1] NATIONAL RESEARCH FELLOW.

[2] Duane and Stenström, *Proc. Nat. Acad. Sci.*, **6**, 477 (1920).

[3] Duane and Patterson, *Ibid.*, **6**, 518 (1920); and *Ibid.*, **8**, 85 (1922).

[4] Duane and Hu, *Physic. Rev.*, **14**, 369 (1919).

[5] A. Sommerfeld, *Ann. Physik*, **76**, 284 (1925); Coster and Goudsmit, *Naturwissenschaften*, **1**, 13 (1925).

[6] Burger and Dorgelo, *Zs. Physik*, **23**, 258 (1924), etc.

[7] Siegbahn and Záček, *Ann. Physik*, **71**, 187 (1923).

[8] E. C. Stoner, *Phil. Mag.*, **48**, 719 (1924).

[9] *Physic. Rev.*, **25**, 882 (1925).

[10] E. C. Unnewehr, *Physic. Rev.*, **22**, 529 (1923).

[11] M. Siegbahn, *Spektroskopie der Röntgenstrahlen*, p. 102.

[12] G. Wentzel, *Ann. Physik*, **66**, 437 (1921).

[13] C. B. Crofutt, *Physic. Rev.*, **24**, 9 (1924).

[14] *Physic. Rev.* (in print).

X-RAYS SCATTERED BY MOLYBDENUM

BY P. A. ROSS

STANFORD UNIVERSITY

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In studying scattered X-rays by means of the ionization spectrometer it becomes essential to obtain the greatest possible X-ray intensity at the scattering body. Small diameter X-ray tubes have been tried and found to be very short lived and unsatisfactory. The alternatives seem to be either excessive energy input or placing the scatterer within the tube close to the focal spot. The scatterer might be mounted upon the cathode,

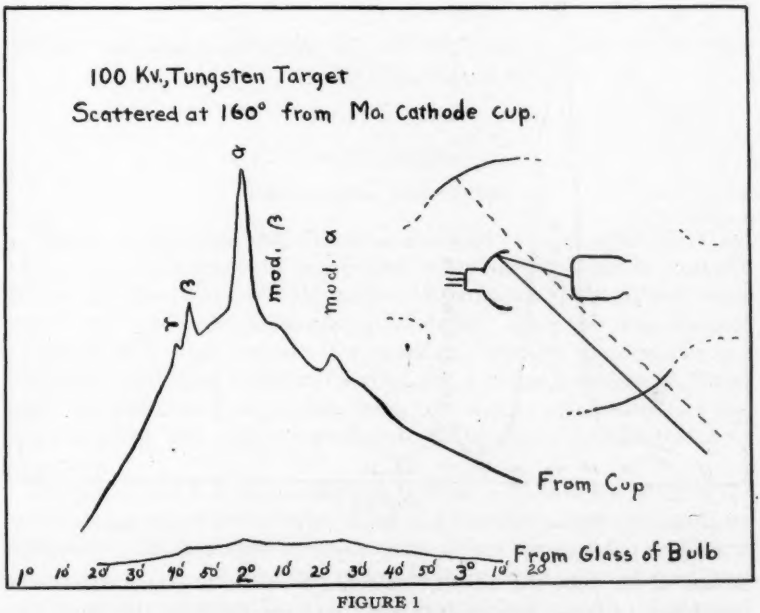
100 Kv., Tungsten Target
Scattered at 160° from Mo. Cathode cup.
σ
τ
mod. β
mod. σ
From Cup
From Glass of Bulb
1° 10′ 20′ 30′ 40′ 50′ 2° 10′ 20′ 30′ 40′ 50′ 3° 10′ 20′

FIGURE 1

the bulb made with the cathode neck removable by a ground glass joint and the tube run while on the pump. As a preliminary to this a test was made of the radiation scattered from the molybdenum shield of the cathode of a standard tungsten tube and of a water-cooled molybdenum tube.

The results are of interest because they are apparently the first *ionization*

measurements of the Compton shift from scattering elements of high atomic weight and because they indicate the probable success of this method in determining the law of variation in intensity of the lines with relation to atomic number.

The X-ray tube was in each case so adjusted that the line of slits passed through the glass of the bulb within the shadow of the target, cleared the target by a small margin and received radiation almost solely from the inside of the cathode cup. As a check experiment, by a small lateral

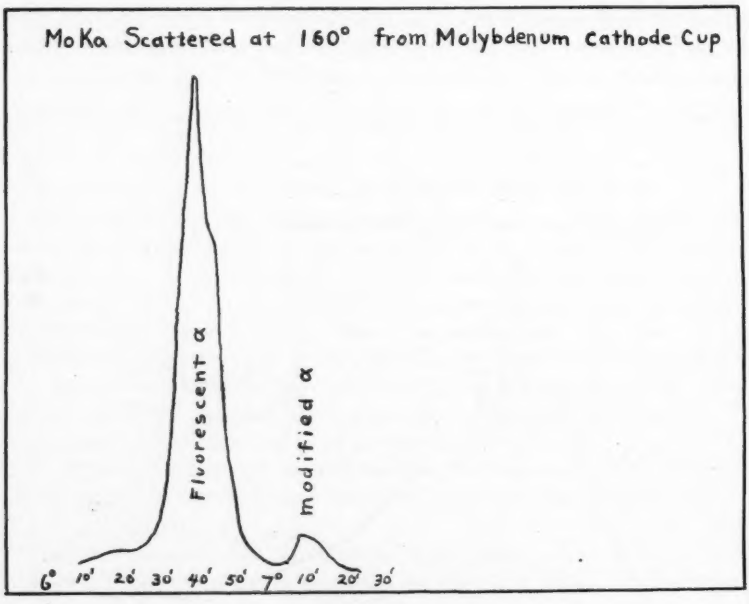

FIGURE 2

motion of the tube the line of slits could be made to pass between the target and cathode, missing both and receiving radiation only from the glass of the bulb. This radiation was found to be small compared with that from the target cup.

Figure 1 shows the results in the case of the tungsten radiation scattered from molybdenum at 160°. The measured shift was 0.047A ± 0.002A and the computed value 0.0469A. In this case the tube was a standard medium focus Coolidge tube driven on 100,000 volts D.C. at 5 milliamperes. The ordinates are electrometer deflections in 10 seconds. The modified

lines in this case are more intense, relative to the unmodified, than would be expected from previous work, the approximate ratio being 9 to 1. As the tube had had hard usage and the cathode cup was much blackened, this probably abnormal ratio may have been due to the fluorescence of a thin film of tungsten evaporated from the filament or focal spot and deposited upon the cup.

Figure 2 shows a similar curve obtained with a water-cooled molybdenum tube ran at 50 Kv. 10 mc. Here intense fluorescence, as would be expected completely masks the unmodified line. The shift again agrees with the computed value well within the limits of error.

Measurements were also made with each tube at 110° scattering angle and the shift found to be 0.035A ± 0.002A.

RATIO OF INTENSITIES OF UNMODIFIED AND MODIFIED LINES IN SCATTERED X-RAYS

BY P. A. ROSS

STANFORD UNIVERSITY

Communicated August 3, 1925

One of the questions that should be answered as soon as possible is that of the ratio of intensity of unmodified to the modified line in scattered X-rays and the variation of this ratio with frequency of the incident beam and atomic number of the scattering substance. Once this ratio has been measured with some accuracy it may indicate whether the electrons in a particular orbit must stand still and scatter, perhaps according to Thomson's classical theory, or are free enough to acquire momentum in a forward direction and scatter according to the quantum laws as stated by Compton.

For this reason it seemed important to make a photometric study of the spectrograms taken by the writer alone and in collaboration with Professor Webster. Although these measurements cannot be expected to give as satisfactory values of the ratio of intensities as might be obtained from careful measurements with an ionization spectrometer, still the range of atomic weights that have been studied photographically is much greater than can be studied by the ionization spectrometer method. It is hoped that by combining these results with those of Compton and others some relations of importance may be brought out.

The photometer used was a modification of the one used by Dr. Harrison employing a Boy's radiomicrometer as the energy measuring device. Deflections were read with a telescope on a scale plotted with logarithms of

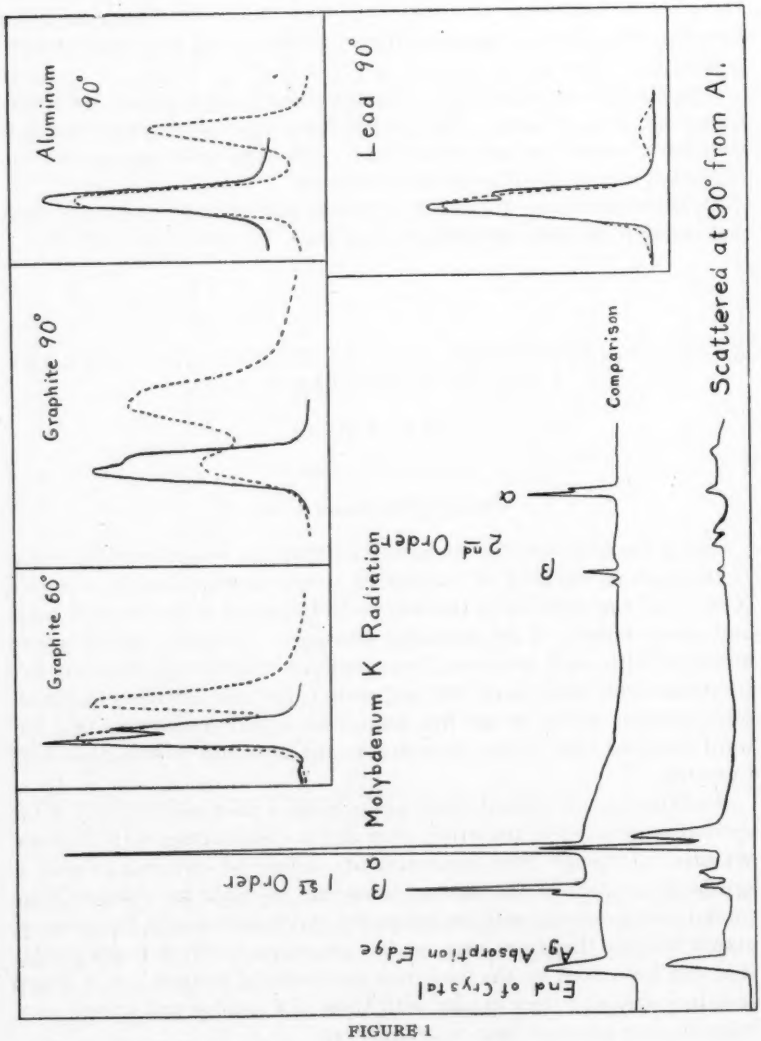
Aluminum 90°
Lead 90°
Graphite 90°
Graphite 60°
Molybdenum K Radiation
2nd Order
1st Order
Comparison
Scattered at 90° from Al.
Ag Absorption Edge
End of Crystal

FIGURE 1

reciprocals. The 0 of the scale came at 100 cm. deflection and by means of an irris diaphragm the fog reading might be made to correspond to this and the scale then gave photographic densities direct. Since, for X-rays,

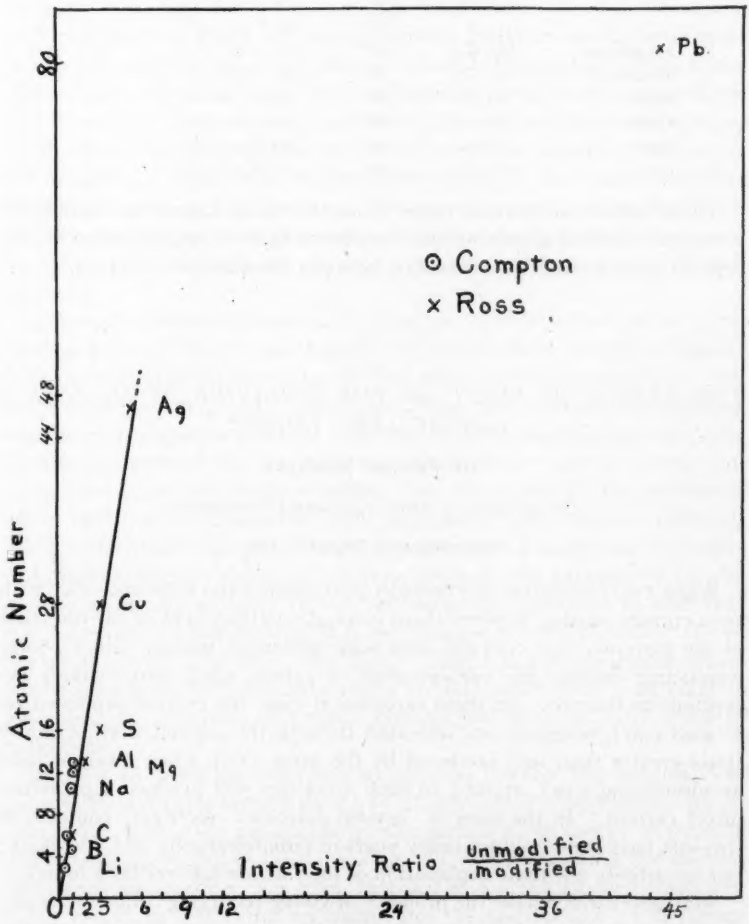

FIGURE 2

the density-log IT curve, except for high densities, is very nearly a straight line passing through the origin, the determination of approximate relative intensities was a very simple matter.

A few of the densitometer curves are shown in figure 1 and the results collected in the following table.

SCATTERING SUBSTANCE	INCIDENT RADIATION	SCATTERING ANGLE	SHIFT (10^{-3} A)	RATIO OF INTENSITIES
Graphite	Mo Kα	30°	3.0 ± 1.0	5.0
		60°	12.0 ± 0.1	1.05
		90°	23.6 ± 0.3	0.59
Aluminum	Mo Kα	90°	24.4 ± 0.1	1.4
Aluminum	Mo Kβ	90°	24.2 ± 0.1	1.0
Sulfur	Mo Kα	90°	24.0 ± 0.5	3.4
Copper	Mo Kα	90°	24.4 ± 0.2	3.1
Silver	Mo Kα	90°	23.8 ± 0.2	4.8
Lead	Mo Kα	90°	24.0 ± 0.2	42.0

These ratios and certain ratios computed from Compton's published curves are plotted against atomic number in figure 2, which indicates, except for lead, a rough linear relation between the quantities plotted.

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*THE EFFECT OF LIGHT ON THE BEHAVIOR OF SELENIUM CONTACT RECTIFIERS**

BY ERNEST MERRITT

DEPARTMENT OF PHYSICS, CORNELL UNIVERSITY

Communicated August 7, 1925

When two conductors are brought into contact the resistance met with by a current passing between them is usually independent of the direction of the current. But contacts with some materials, usually rather poorly conducting crystals like carborundum or galena, often form striking exceptions to the rule. In these exceptional cases the current produced by a small e.m.f. acting in one direction through the contact may be many times greater than that produced by the same e.m.f. when reversed; and an alternating e.m.f. applied to such a contact will produce a pulsating direct current. In the form of "crystal detectors" rectifying contacts of this sort have been used for many years in radiotelegraphy and telephony; but no entirely adequate explanation of their action has yet been found.

Substances which have the property of giving rectifying contacts usually show high thermoelectric power; and in the great majority of cases the rectified current is opposite in direction to the current produced by heating the contact.[1] But high thermoelectric power is not the only factor involved; bismuth and antimony, in spite of their high thermoelectric power, behave like the common metals. The action evidently depends upon the conditions that exist at the surface of a conductor and particularly

upon such conditions as affect the passage of electrons through the surface. The study of rectifying contacts appears therefore to offer a promising means not only of obtaining a clearer understanding of thermoelectric phenomena but also of studying the electrical conditions at the surface of crystals.

The substances which act best as crystal detectors are in most cases compounds and are often minerals of somewhat uncertain composition. The only elements that have been used extensively as detectors are silicon and tellurium. Since it seems probable that the conditions determining rectification, which are obscure at the best, will be simpler in the case of the elements, I have been interested in trying to find other rectifying elements in addition to these two. The present communication deals with the behavior of contacts between selenium and other conductors, and especially with the influence of light upon the rectifying action of such contacts.

Although selenium contacts do not appear to have been used for rectification or for the detection of electric oscillations, there were many reasons for expecting this substance to act like other rectifying materials. For example it has frequently been found that the resistance of ordinary selenium cells depends upon the direction of the current through the cell.[2] Brown[3] has pointed out also that substances which form rectifying contacts show many of the peculiarities that are exhibited by substances whose resistance is changed by light, and many light sensitive materials are known which also give rectifying contacts. Finally, the unusually high thermo-electric power of selenium suggests the probability of well marked rectifying action. It was not surprising therefore to find that selenium contacts show as complete rectification as is exhibited by any of the usual crystal detectors. The high resistance of the selenium contacts and the tendency of selenium surfaces to deteriorate with time make it seem unlikely, however, that these contacts will prove of much value in radio work.

In order to make the resistance as small as possible the selenium was used in the form of a layer from 0.1 to 0.5 mm. thick supported on aluminum. The melted Se was spread on the surface as uniformly as possible when the temperature of the Al was just a few degrees above the melting point and the specimen was then annealed at a constant temperature of either 190°C. or 200°C. for an hour. For the preparations of the specimens used, which were surprisingly uniform in their thickness and light sensitiveness, I am indebted to Dr. R. M. Holmes. Since Se tends to absorb moisture from the air the specimens were kept in a dessicator over $CaCl_2$ when not in use.

The contacts whose rectifying action was to be studied were in some cases made by pressing against the surface the rounded end (radius of

curvature about 0.5 mm.) of a steel or brass rod. Usually, however, knife-edge contacts were used, the metal being filed down to a sharp edge about 1 cm. long and then held against the Se surface by a spring. Except that the knife edge contacts had lower resistance and were less likely to change their adjustment no difference in the behavior of the two was observed. The contact was mounted on an elastic support to prevent disturbances due to mechanical jars. Immediately before making measurements the selenium surface and the knife edge were cleaned with fine emery paper.

Curves showing the relation between applied e.m.f. and current through the contact are shown in figs. 1–4. They are the same type as the characteristics that have been observed in the case of other rectifying materials.

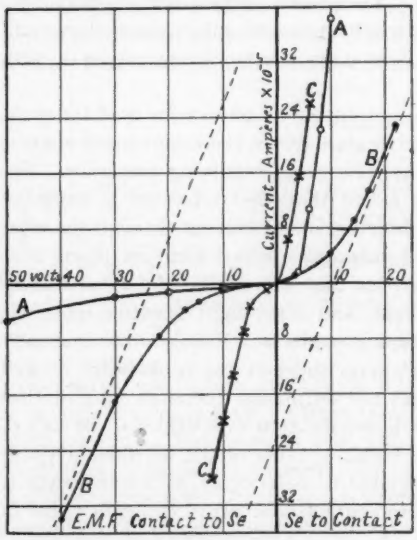

FIGURE 1

Characteristic curves for selenium contacts. A. A knife edge contact of aluminum. B. A knife edge contact of block tin. C. Tin foil folded and pressed against the Se surface.

In the case of the selenium specimens used it was unnecessary to hunt for sensitive points. All portions of the surface seem to be sensitive. Apparently the chief causes of the differences between the characteristics for different contacts are the accidental or intentional variations in pressure and variations from point to point in the thickness and therefore the resistance of the selenium film.

Thus far the metals used for the contact points (or knife edges) have been brass, iron, aluminum, zinc, copper, lead and gold. The resistance of the contact was widely different for the different metals, being least for copper and gold, and there appear to be slight differences in the characteristics obtained with different metals which I hope to discuss in a later paper. But all of the contacts studied behave alike so far as their main features are concerned. For example, the resistance of the contact was always least for a current flowing from the Se to the contact point. This is opposite to the direction in which the thermo-current flows when the contact is warmed.

The appearance of the characteristics usually suggests that for large

values of the e.m.f. the curve approaches a linear asymtote. Although this is most obvious in the case of the positive branch of the curve it seems to be also true for the negative branch provided the curve is followed far enough. Unfortunately this is usually not possible, for at high e.m.f.'s either the contact breaks down or the readings become uncertain because of heating. Curve A of figure 1, for example, whose positive branch became practically straight at 5 volts, still showed considerable curvature at 50 volts when the current was reversed. But when the e.m.f. was increased still further some permanent change occurred so that the curve could not be repeated. In Curves B and C of figure 1, more favorable conditions were met with; in the case of these curves, and in other cases not here plotted, not only does each branch of the characteristic approach a linear asymtote, but the two asymtotes are parallel. Dotted lines have been drawn to show the estimated position of the asymtotes for Curve B.

In studying the effect of light a 400 watt gas filled lamp placed at a distance of about 30 cm. from the contact was used as a source. A water cell 2 cm. thick was usually placed in the path of the beam. In order that the surface of contact might not lie in the shadow of the knife edge the beam was caused to fall obliquely on the surface. In some cases two lamps were used on opposite sides of the knife edge so as to eliminate the shadow altogether. No difference in behavior was observed, however, in this case, except such as would naturally result from the more intense illumination. The water cell appeared to have no influence except to reduce the effect of illumination by about 30%.

For an applied e.m.f. of 0.1 volt or more across the contact the effect of illumination was always to increase the current. But for very small e.m.f.'s and low resistance contacts illumination often resulted in a *decrease* in the current from knife edge to Se, while still causing an increase in the current in the opposite direction. In these contacts light caused a current to flow from Se to knife edge even without any applied e.m.f. This effect was most conveniently studied when copper knife edges were used, since these gave a lower resistance contact than any of the other metals used. It was found that a current was produced by illumination—without any applied e.m.f.—even when the contact itself was in shadow, but that the current became rapidly smaller as the shadow extended further from the contact and could not be detected if the illuminated part of the Se surface was as separated by as much as 1 cm. from the contact. The current flowed from Se to copper. If the contact was heated by warming the copper a current was produced in the opposite direction.

Although the current produced by illumination was much greater in the low resistance contacts, the e.m.f., as measured by means of a potentiometer, was of the same order of magnitude in the case of all the contacts in which this effect was observed, being usually from 1 to 10 milli-

volts. The e.m.f. increased with increasing intensity of illumination. The effect was cut down by about 30% by filtering the light through a 2 cm. water cell and by about 70% when the rays passed through thin black rubber. A small but definite effect remained when water cell and black rubber were both interposed. It seems likely that the effect is due to some action at the surface of contact between Se and Al produced by rays that are able to pass through the thin Se layer. An e.m.f. of this sort has been observed with thick layers of Se by illumination through a semitransparent electrode.[2] In the present case the active rays seem to be in the extreme red and the near infra-red.

Curves showing the relation between current and e.m.f., with and without light, are shown for different contacts in figures 2, 3 and 4. Although the magnitude of the change produced by illumination was widely different for different contacts, the percentage change was in all cases least for the

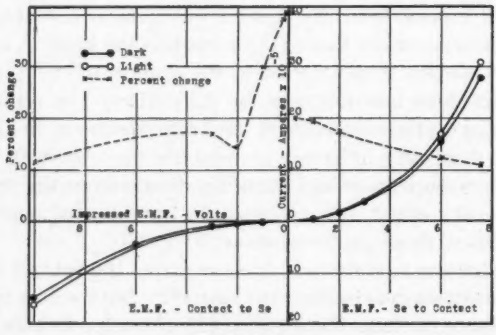

FIGURE 2
Effect of light in the case of an aluminum knife edge contact.

positive branch of the characteristic, i.e., when the current flowed from the selenium to the knife edge. The absolute change for a given e.m.f. was, however, not widely different for the two branches and in fact was usually a little greater for the positive branch of the curve. It will be noticed that the percentage change increases, sometimes quite rapidly, for small values of the impressed e.m.f.; and there is some suggestion of an approach to a limiting value of the percentage change, the same for both branches of the curve, as the applied e.m.f. is increased. There is no great difference in these respects between contacts which rectify only slightly (fig. 3) and those which show a marked difference between the two branches of the characteristics (fig. 4); or between contacts which show slight sensitiveness to light and those in which the change is much greater (figs. 2 and 4). The e.m.f. produced by light in some contacts, and referred to in the pre-

ceding paragraph, is entirely too small to account for the increase observed in the percentage light effect for small impressed e.m.f.'s; in fact for the negative branch of the characteristic it would tend to produce a change just opposite to that observed.

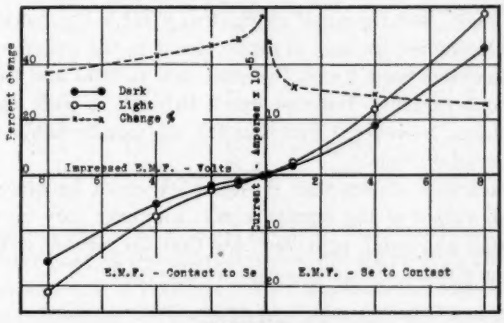

FIGURE 3
Effect of light in the case of an iron knife edge contact.

Owing to the high specific resistance of selenium the resistance is by no means localized in the contact itself. A considerable part of the change due to light is undoubtedly due to the diminution produced by light in the resistance of the mass of the material. But if this were the only cause

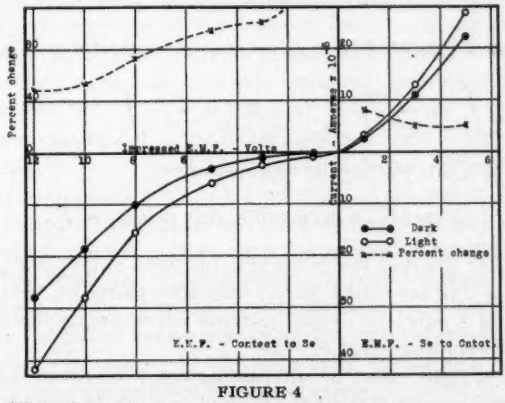

FIGURE 4
Effect of light in the case of a block tin knife edge contact.

of the change in current we should expect the percentage change to be independent of the direction of the current and more nearly constant. It seems clear that light has an effect upon the conditions which cause rectification.

When an alternating e.m.f. was used the results were in most cases such as might be anticipated from the form of the direct current characteristic. A galvanometer in series with the contact indicated current in the direction selenium to contact metal and this rectified current increased, at first rapidly and then more slowly, as the alternating e.m.f. was increased. Austin[4] has found that for small alternating e.m.f.'s the rectified current, in the contacts studied by him, is proportional to the square of the e.m.f. Other experimenters have found the same law to hold and for some contacts the proportionality remains exact through a wide range. With selenium contacts, however, I find that the law usually holds only for e.m.f.'s of the order of 0.1 volt.

It may readily be shown that Austin's law must be obeyed, for sufficiently small values of the applied e.m.f. whatever may be the relation between current and e.m.f. provided only that the current is a continuous function of the e.m.f. For in this case we may put

$$i = f(E) = f(o) + Ef'(o) + \frac{E^2}{2} f''(o) + \ldots$$

Since $i = 0$ for $E = 0$ this may be written

$$i = aE + bE^2 + \ldots$$

and if $E = E_0 \cos pt$

$$i = aE_0 \cos pt + bE_0^2 \cos^2 pt + \ldots$$

The current I indicated by a direct current ammeter will be

$$I = \frac{1}{T}\int_0^T idt = \frac{aE_0}{T}\int_0^T \cos ptdt + \frac{bE_0^2}{T}\int_0^T \cos^2 ptdt + \ldots$$

Since the first integral on the right vanishes

$$I = kE_0^2 + \text{terms in higher powers of } E_0.$$

The fact that Austin's law is obeyed, for small values of E_0, in the case of all contacts thus far tested merely indicates, therefore, that the current is a function of E only—i.e., that hysteresis effects are negligible—and that the function is continuous at $E = 0$.

The effect of light, when an alternating e.m.f. was used was ordinarily to increase the magnitude of the rectified current, as would be expected from the fact that in the characteristics shown in figures 2, 3 and 4 the increase in current produced by light is greater for the positive branch of the curve. But for some contacts the rectified current was *diminished* by illumination. Such anomalous behavior is also to be expected, for among the many characteristics recorded, although not among those here

plotted, some were found in which not only the relative increase due to light but also the absolute increase was greater for the negative branch of the curve. The data for a contact of this sort are given in Table I.

TABLE I

E. M. F. ACROSS CONTACT	CURRENT, CONTACT TO SE, $\times 10^8$			CURRENT, SE TO CONTACT, $\times 10^8$		
	IN DARK	INCREASE BY LIGHT	PERCENTAGE INCREASE	IN DARK	INCREASE BY LIGHT	PERCENTAGE INCREASE
0.10	1.72	0.36	21.0%	2.25	0.41	18.2%
0.19	3.4	0.72	21.3	5.0	0.69	13.8
0.47	9.3	2.03	21.7	14.7	1.8	12.3
1.5	42.5	7.4	17.4	66.0	6.6	10.0
2.5	85.5	14.8	17.4	118.0	12.5	10.6
3.5	141.0	23.3	16.5	186.0	17.8	9.6
4.5	210.0	36.0	17.2	253.0	26.7	10.5

This contact is also interesting on account of the fact that while throughout the greater part of the characteristic the effect of light was least for the direction selenium to contact, yet for small e.m.f.'s the light effect was *greater* in this direction. For an alternating e.m.f. of 0.1 volt this contact gave a rectified current from selenium to copper of 1.4×10^{-9} amp. and this was increased to 1.57×10^{-9} by illumination. For an e.m.f. of 1. volt the same contact gave a rectified current of 585×10^{-9} amp. also in the direction Se to Cu, and this was diminished to 580×10^{-9} by light. At 5 v. the rectified current was $18{,}200 \times 10^{-9}$ in the dark and 16,500 when illuminated. Although this contact showed relatively small light effect and rectification it was extremely stable and the observations could be repeated in any sequence with consistent results. Many contacts were found in which the negative light effect was much greater; but these highly sensitive contacts were quite unstable and gave results that could not be reproduced.

With an alternating e.m.f. the phenomena are likely to be quite complicated and they are in all cases less capable of definite interpretation than are the results obtained with direct current. This fact is illustrated by the curves of figure 5 where the rectified current with and without light is plotted against the a.c. voltage for a copper knife edge contact. It will be noticed that while the rectified current is from selenium to copper for e.m.f.'s below 2.7 volts, it is in the opposite direction for higher voltages. For points to the left of the reversal point the rectified current is increased by illumination; for points to the right it is diminished. Between 2.7 volts and 3.7 volts the rectified current is in one direction in the light and in the opposite direction in the dark. The light effect as indicated by the algebraic difference between the rectified currents with and without illumination is plotted as a broken line in figure 5 and increases steadily with increasing e.m.f.

A reversal of rectification similar to that shown in figure 5 has been observed by Austin[4] in the case of a contact between Tellurium and

Aluminum. It may be explained, as suggested by Austin, by the increased heating of the contact at higher current densities. In the case of selenium contacts this explanation is made especially plausible by the fact that the

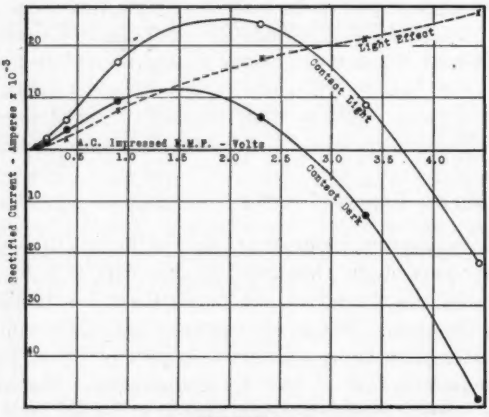

FIGURE 5
Effect of light on the rectified current through a rounded brass point contact.

direction of the rectified current may often be reversed simply by increasing the force pressing the knife edge against the selenium, which allows more current to flow and therefore brings about greater heating.

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THE ACOUSTIC PRESSURE IN TUBES CAPPED BY HIGH RESISTANCE TELEPHONES, VIBRATING IN DIFFERENT PHASES*

BY CARL BARUS

DEPARTMENT OF PHYSICS, BROWN UNIVERSITY

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1. *Apparatus.*—As in the preceding papers (these PROCEEDINGS, May, July), the acoustic pressure is measured by pin hole probes at the middle of the tube connecting the telephones, communicating with the interferometer U-tube. The fringe displacement is s. In dealing with larger inductances the radio telephones are clearly more useful. Furthermore an enlargement of the end connections of the pipe joining the paired telephones suggested itself. Hitherto the junction had been made with perforated rubber stoppers and quill tubes connectors. In the present case the tube ends were attached to the telephone mouth pieces directly, in order to introduce the least obstruction possible. The result, however, was but a slight rise in pitch from the former a', to $^{\flat}b'$ in the present adjustment.

Figure 1 gives evidence of the extreme sharpness of the resonance crests for the summational position of the switch S (phase). Below e', however, there is multiresonance which is difficult to construe, as a definite $^{\flat}b'$ crest could not be found unless the diffuse c' crest here replaces it. The occurrence of the sharp strong $^{\flat}b''$ crest must have had an analogue in the preceding work, though for some reason it was not detected. It may have been deadened by the quill tube connectors.

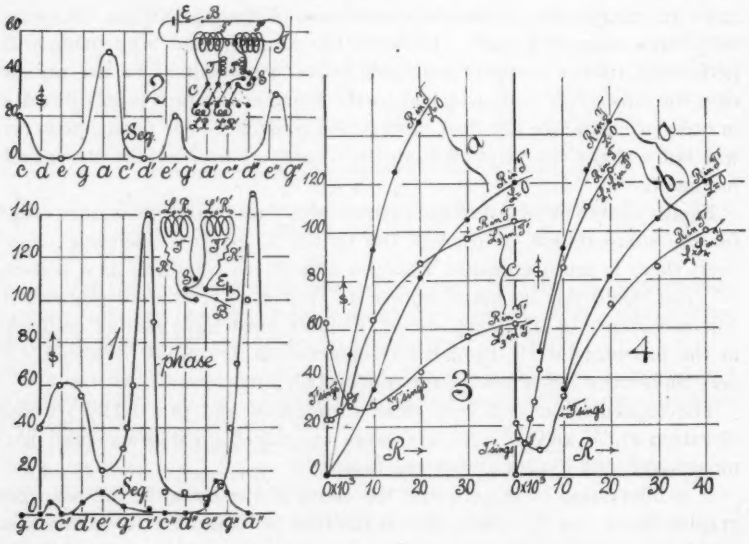
The sequence curve is very weak throughout and practically without elevation at $^{\flat}b'$ and $^{\flat}b''$. It has, however, picked up its own small harmonics at f' and f'' with a little one near b.

It is interesting to inquire into the cause of the crests in the sequence graphs, figure 1 or 2. Since the f is the fifth of $^{\flat}b$ and the telephone note is rich in overtones, one may surmise that in the sequence graph, while the $^{\flat}b$ is eliminated, the f overtone would not be. In fact if we take the usual equations $y = a \sin(\varphi - \varphi_0)$ and $y' = a' \sin(\varphi + \varphi_0')$, the compound harmonic is $A \sin(\varphi - \Phi_0)$, where $\tan \Phi_0 = \Sigma a \sin \varphi_0 / \Sigma a \cos \varphi_0$ and $A^2 = (\Sigma a \sin \varphi_0)^2 + \Sigma(a \cos \varphi_0)^2$. Hence if we put $a = a'$, $\varphi_0 = 0$ and $\varphi_0' = \pi/2$ for the f harmonic, it follows that $\Phi_0 = 45°$ and $A = a\sqrt{2}$, so that the f is not eliminated.

In the sequence graph balanced there is probably no audible fundamental inasmuch as a single wave runs from end to end of the tube without interference.

2. *Advantages of a Small Inductor.*—As it is the chief purpose of the

present adjustment to meet the case of large resistances and inductances, the small inductor I, figure 2 (about 0.35 henry in each secondary I', I''), was inserted as there shown, using the commutating device C for the exchange of auxiliary inductances L, L', etc. Each telephone, T, T', has its own secondary and T' is provided with a switch S. B is the (motor) periodic break. Under these circumstances a single cell at E will throw the crests of the phase graph far out of the field of view. The sequence graph, figure 2, is correspondingly developed with the f crests sharper than in figure 1. The strong b crest, here obtained, is a new feature and there was a further marked development even as low as c. These crests, however, are to be avoided and the measurements made corresponding to the sharp $^{b}b''$ or $^{b}b'$ resonance in the phase graph.



There are two difficulties encountered in making these experiments: The first is the sharpness of the resonance crests, the other the rather slow growth of the maximum displacement.

Examples of typical results are given in figures 3 and 4, which are sequence graphs obtained by the method figure 2. The group a refers to a comparison of external resistances ($R = 0$, $R' = 0$ to 4×10^4 ohms) only; the graph b, to a comparison of $L_1 + L_4$ (0.36 henry and 10 ohms) with R (0 to 4×10^4) and the graph c to L_3 (1.4 henry and 550 ohms) with R. To these must be added the internal resistances R_0 and inductances L_0 of the telephones and coils T, I and T', I', respectively. The

discussion of the curves is complicated by the unequal efficiency of the telephones considered in the last paper, and by the unequal induction E, E' of the two coils. As a result the upper curves of a pair are usually without well developed minima; whereas the lower curves (commutated) often show pronounced minima, on the two sides of which the sound passes apparently from one telephone to the other. This minimum will be determined by the superposition of an R-graph decreasing with R and an L-graph increasing with R. If we place the minimum at their intersection, $s' = s$ cancels out and for currents $i = s/c$, etc., the equation for parallel coupling reduces to, $c^2\{(R + R_0^1)^2 + L'^2_0\,\omega^2\} = c'^2\ \{R_0^2 + (L + L_0)^2\omega^2\}$ and gives a good order of values for the minimum at R if the load is L. One notes that the R position of the minima as well as the intersection of the paired graphs rise with increasing L.

3. *The Intersection of Paired Graphs.*—If we treat the present experiments in which the currents in the T, T' circuits are coupled by mutual induction as if they were ordinary currents in parallel, the results are a close approach to the actual case. The later data of figures 3 and 4 are available for testing the case by computing R for the points I at which the graphs b and c intersect. The equation for the purpose has been given ($\Delta s = \Delta s'$) heretofore and may be put in the form

$$c'/\sqrt{(R' + R_0')^2 + L'^2_0\omega^2} + c/\sqrt{(R + R_0)^2 + L_0^2\omega^2}$$
$$= c'/\sqrt{(R_0' + R_x)^2 + (L_x + L_0')^2\omega^2} + c/\sqrt{(R_0 + R_x)^2 + (L_x + L_0)^2\omega^2}.$$

Here R_x and L_x are the resistance and inductance of the coil commutated and $R' = R$ is the observed resistance at the point of intersection. R_0, L_0, R_0', L_0' are the constants of the secondaries and telephones as indicated in figure 1. The data are, since $\omega = 2934$,

Telephones:	T', $R_0' = 1110$,	$L_0' = 1.2$	T,	$R_0 = 1090$	$L = 1.2$
Secondaries:	I', 30	0.4	I,	32	0.3
Total:	$R_0' = 1140$,	$L_0' = 1.6$	Total	$R_0 = 1122$	$L_0 = 1.5$

These values are nearly enough alike for the T and T' circuits that a mean may be taken tentatively, with the object of simplifying the cumbersome equation just stated. This reduces, if $L = L_0'$, $R = R_0'$ and $R = R'$ at the intersection to

$$(R + R_0)^2 = (R_x + R_0)^2 + \{(L_x + L_0)^2 - L_0^2\}\,\omega^2.$$

If now R_x, L_x refer to the coils $L_1 + L_4$ of the graph b, figure 4, $R_x = 11$ and $L_x = 0.36$, from which $R = 2380$ ohms. The point of intersection in figure 4, b is at about 2000 ohms.

If R_x, L_x refer to the coil L_3 of graph c, figure 3, $R_x = 550$ ohms, $L_3 = 1.4$ henry, from which $R = 6511$ ohms, while the point of intersection in figure 3 c is between 4000 and 5000 ohms.

In both cases therefore the computed point of intersection lies above the point given by the graphs; but the order of values is satisfactory, as one cannot expect sharp values from the approximate current equations $i = s/c$ and $i' = s'/c'$, nor the coupling postulated. Moreover the same equations have been made to include the inequalities of induction E, E'. The equation for $(R + R_0)^2$, shows that the point of intersection of the paired curves moves to the right both with R_x and L_x of the commutated coils.

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